MATHEMATICS

MODEL PAPER – 1

Time : 3 Hours + 15 Minutes] [Total Marks : 100

Instructions to the Candidates :

1. Candidates are required to give their answers in their own words as far as practicable.
2. Figures in the right hand margin indicate full marks.
3. 15 Minutes of extra time has been allotted for the candidates to read the questions carefully.
4. This question paper is divided into two sections—**SECTION – A** and **SECTION – B**.
5. In **SECTION – A** there are **100 Objective Type Question,** out of which only 50 objective question be answered. Darken the circle with blue/black ball pen against the correct option on OMR Sheet provided to you. Do not use **Whitener/Liquid/Blade/Nail** on OMR paper; otherwise the result will be invalid.
6. In **SECTION – B**, there are **30 Short Answer Type Questions** (each carrying 2 marks), out of which any 15 questions are be answered.

   Apart from this, there are **8 Long Answer Type Question** (Each Carrying 5 marks), out of which 4 Questions are to be answered.
7. Use of any electronic device is prohibited.

SECTION – A : Objective Type Questions

Direction : *There are 100 Objective Type Questions, out of which only 50 objective questions to be answered. Mark the correct option on the OMR Answer Sheet.* 50 × 1 = 50

1. If $A = \{a, b\}$, $B = \{1, 2, 3\}$ then total number of one-one functions from A to B is :

(A) 6 (B) 8
(C) 9 (D) None of these

2. $x \in R, \cot^{-1} x =$

(A) $\frac{\pi}{2} - \sin^{-1} x$ (B) $\frac{\pi}{2} - \cos^{-1} x$
(C) $\frac{\pi}{2} - \tan^{-1} x$ (D) $\frac{\pi}{2} - \sec^{-1} x$

3. If A and B are independent event $P(A) = 0.3$ and $P(B) = 0.4$ then $P(A \cap B) =$

(A) 0.12 (B) 0.21
(C) 0.75 (D) 0.7

4. The maximum value of $z = 6x + 3y$ subject to constraints $x + y \leq 25$, $\begin{matrix} x \geq 0 \\ y \geq 0 \end{matrix}$ is :

(A) 150 (B) 225
(C) 425 (D) None of these

5. If the direction ratios of two parallel lines are a_1, b_1, c_1 and a_2, b_2, c_2 then $\frac{a_1 c_2}{a_2} =$

(A) b_1 (B) b_2
(C) b_3 (D) c_1

6. $(\hat{j} - 2\hat{i}) \cdot (\hat{k} + 3\hat{i} - \hat{j}) =$

(A) 0 (B) –6
(C) –7 (D) 8

7. $\vec{k} \cdot (\vec{i} + \vec{j}) =$

(A) 0 (B) 1
(C) 2 (D) –1

8. The solution of the differential equation $x^2 dx + y^2 dy = 0$ is :

(A) $x^3 + y^3 = k$ (B) $x^2 + y^2 = k$
(C) $x^2 - y^2 = k$ (D) $x^3 - y^3 = k$

9. $\int_{-\pi/6}^{\pi/6} \tan \theta \, d\theta =$

(A) 0 (B) 1 (C) 2 (D) 3

10. $\int \sin^3 \theta \operatorname{cosec}^2 \theta \, d\theta =$

(A) $c + \theta$ (B) $c + \cos\theta$
(C) $c - \cos\theta$ (D) $c + \sin\theta$

11. $\frac{d}{dx}\left(e^x + \cos 5x\right) =$

(A) $e^x + \cos 5x$ (B) $e^x + 5\sin 5x$
(C) $e^x - 5\sin 5x$ (D) $e^x - 5\cos 5x$

12. $\frac{d}{dx}(\sin 2x + e^x - \cos x) =$

(A) $\cos 2x + e^x - \sin x$ (B) $2\cos 2x + e^x + \sin x$
(C) $2\cos 2x + e^x - \sin x$ (D) $-2\cos 2x + e^x + \sin x$

13. $\begin{vmatrix} 1 & 1 & 5 \\ 4 & 9 & 17 \\ 5 & 10 & 22 \end{vmatrix} =$

(A) 264 (B) 1221 (C) 0 (D) 1

14. $\begin{vmatrix} -\sin\theta & \cos\theta \\ \sec\theta & \text{cosec}\,\theta \end{vmatrix} =$

(A) 0 (B) –1 (C) –2 (D) – sin 2θ

15. How many distinct relations can be defined on the set $A = \{1, 2, 3\}$?

(A) 2^9 (B) 2^3 (C) 9 (D) 2^6

16. $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} =$

(A) $\tan^{-1}\frac{44}{29}$ (B) $\frac{\pi}{2}$ (C) 0 (D) $\frac{\pi}{4}$

17. The principle value of $\sin^{-1}\frac{\sqrt{3}}{2}$ is :

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

18. If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then

(A) $\vec{a} \parallel \vec{a}$ (B) $\vec{a} \perp \vec{b}$

(C) $|\vec{a}| = |\vec{b}|$ (D) None of these

19. $\int x^5 dx$

(A) $\frac{x^6}{6}+k$ (B) $\frac{x^5}{5}+k$ (C) $\frac{x^7}{7}+k$ (D) $\frac{x^8}{8}+k$

20. $\int o dx$

(A) K (B) 0 (C) 1 (D) –1

21. If $x = \sin\theta$, $y = \cos\theta$, then $\frac{dy}{dx} =$

(A) tan θ (B) – tan θ

(C) cot θ (D) None ot these

22. $\frac{d}{dx}(\sqrt{x}) =$

(A) $2\sqrt{x}$ (B) $\frac{1}{2\sqrt{x}}$ (C) $\frac{\sqrt{x}}{2}$ (D) $\frac{1}{\sqrt{x}}$

23. If $\theta + \phi = 90°$ then $\begin{vmatrix} \cos\theta & \sin\theta \\ \sin\phi & \cos\phi \end{vmatrix} =$

(A) 1 (B) 0 (C) –1 (D) ∞

24. If matrix B is the inverse matrix of A then $AB = BA =$

(A) A (B) B

(C) I (unit matrix) (D) Null matrix

25. $P(A \cup B) =$

(A) $P(A) + P(B) + P(A \cap B)$ (B) $P(A) - P(B) - P(A \cap B)$

(C) $P(A) + P(B) - P(A \cap B)$ (D) $P(A) - P(B) + (A \cap B)$

26. If $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(A \cup B) = \frac{5}{8}$ then $P|A \cap B) =$

(A) $\frac{3}{8}$ (B) $\frac{1}{8}$ (C) $\frac{2}{8}$ (D) $\frac{5}{8}$

27. The equation of the xy-plane is :

(A) $x = 0, y = 0$ (B) $z = 0$

(C) $x = y \neq 0$ (D) None of these

28. If two planes $2x - 4y + 3z = 5$ and $x + 2y + \lambda z = 12$ are perpendicular to each other, then $\lambda =$

(A) –2 (B) 2

(C) 3 (D) None of these

29. $\int_4^9 \sqrt{x}\, dx = ?$

(A) $\frac{38}{3}$ (B) $\frac{35}{3}$ (C) $\frac{36}{7}$ (D) $\frac{45}{3}$

30. $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} =$

(A) $(x - y)(y + z)(z + x)$ (B) $(x + y)(y - z)(z - x)$

(C) $(x - y)(y - z)(z + x)$ (D) $(x - y)(y - z)(z - x)$

31. The value of the determinant :

$\begin{vmatrix} 3 & 1 & 7 \\ 5 & 0 & 2 \\ 2 & 5 & 3 \end{vmatrix} =$

(A) 124 (B) 125 (C) 134 (D) 144

32. The equation of a plane parallel to the plane $5x - 6y + 7z - 8 = 0$ is :

(A) $5x - 6y + 7z + 5 = 0$ (B) $6x - 7y + 7z - 8 = 0$

(C) $5x + 6y - 7z - 8 = 0$ (D) None of these

33. $\vec{k} \times (\vec{i} \times \vec{j}) =$

(A) $\vec{0}$ (B) $\vec{i}$ (C) $\vec{j}$ (D) $\vec{k}$

34. If $|\vec{a}| = 2$ and $\lambda\vec{a}$ is a unit vector then λ has the value :

(A) 1 (B) $\frac{1}{2}$

(C) 2 (D) None of these

35. The solution of $\frac{dy}{dx} = 1 + x + y + xy$ is :

(A) $x - y = k(1 + xy)$ (B) $\log(1 + y) = x + \frac{x^2}{2} + k$

(C) $\log(1+x) + y + \frac{y^2}{2} = k$ (D) None of these

36. If $y = \log x^x$, then $\frac{dy}{dx} =$

(A) 1 (B) $\log x$

(C) $\log(ex)$ (D) None of these

37. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ is :

(A) $\frac{y}{\sqrt{1+x^2}}$ (B) $-\frac{x}{\sqrt{1+x^2}}$

(C) $\frac{x}{\sqrt{1-x^2}}$ (D) None of these

38. $\sec^{-1}x + \text{cosec}^{-1}x =$

(A) π (B) 0 (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

39. $\tan^{-1}(-x)$

(A) $\tan^{-1} x$ (B) $-\tan^{-1} x$

(C) $\pi - \tan^{-1}\frac{1}{x}$ (D) $\sin^{-1} x - \pi$

40. $\int_0^1 \frac{dx}{1+x^2} = ?$

(A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $-\frac{\pi}{2}$

41. $\int \frac{e^x}{x}(1 + x\log_e x)dx =$

(A) $\frac{e^x}{x} + c$ (B) $e^x \log x + c$

(C) $\log x + c$ (D) $\frac{e^x}{x}\log x + c$

42. If $\vec{a} = \vec{i} + \vec{j} + 3\vec{k}; \vec{b} = 2\vec{i} + 3\vec{j} - 5\vec{k}$ then $\vec{a} \bullet \vec{b}$ =

(A) 10 (B) –10 (C) 20 (D) 5

43. If $x > a$, $\int \frac{dx}{x^2 - a^2} =$

(A) $\frac{2}{2a} \log \frac{x-a}{x+a} + k$ (B) $\frac{1}{2a} \log \frac{x+a}{x-a} + k$

(C) $\frac{1}{a} \log (x^2 - a^2) + k$ (D) $\log (x + \sqrt{x^2 - a^2} + k)$

44. $\int \frac{\cos 2x \, dx}{(\sin x + \cos x)^2} =$

(A) $-\frac{1}{\sin x + \cos x} + c$ (B) $\log (\sin x + \cos x) + c$

(C) $\log |\sin x - \cos x| + c$ (D) $\frac{1}{(\sin x + \cos x)^2}$

45. $\begin{vmatrix} 10 & 2 \\ 35 & 7 \end{vmatrix} =$ ......

(A) 4 (B) 0 (C) 3 (D) 6

46. The rate of change of the area of a circle with respect to its radius r at $r = 6$ cm is :

(A) 10π (B) 12π (C) 8π (D) 11π

47. The order of the differential equation $\frac{dy}{dx} + y = e^x$ is :

(A) 2 (B) –1 (C) 1 (D) –2

48. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is :

(A) $e^x + e^{-y}$ (B) $e^x + e^y = K$

(C) $e^{-x} + e^y = K$ (D) $e^{-x} + e^{-y} = K$

49. The operation * is defined as $a * b = 3a + 4b - 2$, then $4 * 5$ is :

(A) 30 (B) 20 (C) 10 (D) 15

50. If $A = \{1, 2, 3\}$, $B = \{6, 7, 8\}$ and $f : A \to B$ is a function such that $f(x) = x + 5$ then what type of a function is f?

(A) into (B) one-one onto

(C) many-one onto (D) constant function

51. $f : A \to B$ Will be an into function if

(A) $f(A) \subset B$ (B) $f(A) = B$

(C) $B \subset f(A)$ (D) $f(B) \subset A$

52. If $f : R \to R$ such that $f(x) = 3x - 4$ then which of the following is $f^{-1}(x)$?

(A) $\frac{1}{3}(x + 4)$ (B) $\frac{1}{3}(x - 4)$

(C) $3x - 4$ (D) undefined

53. $A = \begin{bmatrix} 3 & 6 \\ 5 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 8 \\ 5 & 6 \end{bmatrix} \Rightarrow 2A + 3B =$

(A) $\begin{bmatrix} 27 & 24 \\ 22 & 10 \end{bmatrix}$ (B) $\begin{bmatrix} 27 & 24 \\ 22 & 10 \end{bmatrix}$ (C) $\begin{bmatrix} 27 & 24 \\ 22 & 10 \end{bmatrix}$ (D) $\begin{bmatrix} 27 & 24 \\ 22 & 10 \end{bmatrix}$

54. If a, b, c are in A.P., then the determinant

$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$ is

(A) 1 (B) x (C) 0 (D) $2x$

55. If $y = \sec^{-1}\left[\frac{\sqrt{x}+1}{\sqrt{x}-1}\right] + \sin^{-1}\left[\frac{\sqrt{x}-1}{\sqrt{x}+1}\right]$, then $\frac{dy}{dx}$ is equal :

(A) $\frac{\pi}{4}$ (B) π (C) $\frac{\pi}{2}$ (D) 0

56. $\frac{d}{dx}(\cos^{-1} x) =$

(A) $\frac{1}{2\sqrt{1-x^2}}$ (B) $\sqrt{1-x^2}$ (C) $\frac{-1}{\sqrt{1-x^2}}$ (D) $\frac{1}{\sqrt{1-x^2}}$

57. $\int_0^1 (x)\, dx =$

(A) 0 (B) 1 (C) 2 (D) $\frac{1}{2}$

58. $\int \cot^2 x \, dx =$

(A) $\cot x - x + k$ (B) $2 \cot x \operatorname{cosec}^2 x + k$

(C) $-\cot x - x + k$ (D) $x + \cot x + k$

59. If $\vec{a} = 2\vec{i} - 5\vec{j} + k$ and $\vec{b} = 4\vec{i} + 2\vec{j} + \vec{k}$ then $\vec{a}.\vec{b} =$

(A) 0 (B) –1 (C) 1 (D) 2

60. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ than A^2 is :

(A) 27 A (B) 2A (C) 3 A (D) 1

61. If $a : b$ be odds in favour of event E, then $P(E)$

(A) $\frac{b}{a+b}$ (B) $\frac{b}{a-b}$ (C) $\frac{a}{a+b}$ (D) $\frac{a}{a-b}$

62. $\int \tan x \, dx$

(A) $\log |\sec x| + k$ (B) $\log |\sin x| + k$

(C) $\log |\cos x| + k$ (D) $\log |\operatorname{cosec} x| + k$

63. $\int_0^2 e^{x/2} dx = ?$

(A) $2(e - 1)$ (B) $2(e + 1)$

(C) $2(1 - e)$ (D) $2(e^2 - 1)$

64. $\frac{d}{dx}(e^{x^3}) =$

(A) e^{x^3} (B) $3x^2e^x$ (C) $3x^2e^{x^2}$ (D) $3x^2e^{x^3}$

65. $\frac{d}{dx}(\sin 2x) =$

(A) $\cos 2x$ (B) $\frac{\cos 2x}{2}$

(C) $2 \sin 2x$ (D) $2\cos 2x$

66. $\sin^{-1}\left(\sin\frac{7\pi}{6}\right) =$

(A) $\frac{7\pi}{6}$ (B) $\frac{\pi}{6}$ (C) $\frac{-\pi}{6}$ (D) $\frac{5\pi}{6}$

67. $\sec^2(\tan^{-1} 5) + \operatorname{cosec}^2(\cot^{-1} 5) =$

(A) 10 (B) 50 (C) 51 (D) 52

68. If A is an invertible matrix and A^{-1} is the inverse matrix of A, then :

(A) $A + A^{-1} = I$ (B) $AA^{-1} = I$

(C) $A - A^{-1} = I$ (D) $AA^{-1} = 2I$

69. The solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is :

(A) $e^x + e^{-y} + k = 0$ (B) $e^{2x} = ke^y$

(C) $e^x = ke^{2y}$ (D) $e^x = ke^y$

70. The integrating factor of the differential equation $\frac{dx}{dy} + Px = Q$ is :

(A) $\int_e Pdx$ (B) $\int_e Pdy$ (C) $\int_e Qdx$ (D) $\int_e Qdy$

71. A pair of dice are rolled. The probability of obtaining an even prime number on each die is :

(A) $\frac{1}{36}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) 0

72. If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$ then, $P(A' \cap B') =$

(A) $\frac{13}{24}$ (B) $\frac{13}{8}$ (C) $\frac{13}{9}$ (D) $\frac{13}{4}$

73. The direction cosines of z-axis are :

(A) (0, 0, 0) (B) (1, 0, 0)

(C) (0, 0, 1) (D) (0, 1, 0)

74. Let l_1, m_1, n_2 and l_2, m_2, n_2 be the direction cosines of two straight lines. Both the lines perpendicular to each other, if :

(A) $l_1l_2 + m_1m_2 + n_1n_2 = 0$ (B) $l_1l_2 + m_1m_2 + n_1n_2 = 1$

(C) $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$ (D) $\frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2} = 0$

75. If O be the origin and $A(x, y, z)$ be any point, then $\overrightarrow{OA} =$

(A) $x\vec{i} - y\vec{j} - z\vec{k}$ (B) $x\vec{i} + y\vec{j} - z\vec{k}$

(C) $x\vec{i} + y\vec{j} + z\vec{k}$ (D) $zx\vec{i} + yz\vec{j} + xy\vec{k}$

76. $(\vec{i} + 2\vec{j} + 3\vec{k}) \bullet (2\vec{i} + 3\vec{j} + 4\vec{k}) =$

(A) 14 (B) 16 (C) 18 (D) 20

77. $\int_0^2 (x^2 + 1)\, dx =$

(A) $\frac{8}{3}$ (B) $\frac{14}{3}$ (C) $\frac{13}{3}$ (D) $\frac{1}{3}$

78. The integration of 0 with respect to x is :

(A) 0 (B) k (C) $x + k$ (D) $x^2 + k$

79. $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 2 \end{vmatrix} =$ ......

(A) 5 (B) 17 (C) 8 (D) 0

80. $\frac{d}{dx}(\cot x) =$

(A) $\tan x$ (B) $\operatorname{cosec}^2 x$

(C) $-\operatorname{cosec}^2 x$ (D) $\operatorname{cosec} x \cot x$

81. $\frac{d}{dx}(\sin^2 x) =$

(A) $\sin 2x$ (B) $\cos 2x$

(C) $\tan 2x$ (D) $\cot 2x$

82. $A = \begin{bmatrix} 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}$, $B = \begin{bmatrix} 11 & 10 & 9 \\ 8 & 7 & 6 \end{bmatrix} \Rightarrow 2A + 2B =$

(A) $\begin{bmatrix} 20 & 20 & 20 \\ 20 & 20 & 20 \end{bmatrix}$ (B) $\begin{bmatrix} 40 & 40 & 40 \\ 40 & 40 & 40 \end{bmatrix}$

(C) $\begin{bmatrix} 20 & 20 & 20 \\ 40 & 40 & 40 \end{bmatrix}$ (D) $\begin{bmatrix} 40 & 40 & 40 \\ 20 & 20 & 20 \end{bmatrix}$

83. $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \Rightarrow A^2 =$

(A) 2A (B) A (C) $\frac{1}{2}$A (D) 4A

84. The value of $\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$ is :

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) $\frac{\pi}{4}$

85. The solution set of the equation $\tan^{-1} x - \cot^{-1} x = \cos^{-1}(2 - x)$ is :

(A) [0, 1] (B) [−1, 1]

(C) [1, 3] (D) None of these

86. The distance between the points (4, 3, 7) and (1, −1, −5) is :

(A) 7 (B) 12 (C) 13 (D) 24

87. The direction ratios of two straight lines are l, m, n and l_1, m_1, n_1. The lines will be perpendicular to each other if :

(A) $\frac{l}{l_1} = \frac{m}{m_1} = \frac{n}{n_1}$ (B) $\frac{l}{l_1} + \frac{m}{m_1} + \frac{n}{n_1} = 0$

(C) $ll_1 + mm_1 + nn_1 = 0$ (D) $ll_1 + mm_1 + nn_1 = 1$

88. $\int_{-\pi/2}^{\pi/2} |\cos x|\, dx =$

(A) 2 (B) 1 (C) 0 (D) 4

89. $\int_0^{\pi/2} \log\cot\theta \cdot d\theta =$

(A) $\frac{\pi}{2}\log 2$ (B) $\frac{\pi}{4}\log 2$

(C) $2\pi\log 2$ (D) 0

90. $\vec{i} \times \vec{j} =$

(A) 0 (B) 1 (C) $\vec{k}$ (D) $-\vec{k}$

91. $\vec{k}\cdot\vec{k}=$

(A) 0 (B) 1 (C) $\vec{i}$ (D) $\vec{j}$

92. The solution of $(1+x^2)\,dy=(1+y^2)\,dx$ is :

(A) $x+y=k(1-xy)$ (B) $y-x=k(1-xy)$

(C) $x+y=k(1+xy)$ (D) $y-x=k(1+xy)$

93. $\frac{d}{dx}\left\{\lim_{x\to a}\cos 3x\right\}=$

(A) $-\sin 3x$ (B) 1 (C) $-3\sin x$ (D) 0

94. $\frac{d^3}{dx^3}(x^4)=$

(A) $4x^3$ (B) $12x^2$ (C) $24x$ (D) 24

95. $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}=$

(A) $\cos 2\theta$ (B) 1 (C) 0 (D) -1

96. If $A=\begin{bmatrix}\lambda & 0\\ 1 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$ and $A^2=B$, then $\lambda=$

(A) -1 (B) 1

(C) 4 (D) None of these

97. The chance of getting a doublet with 2 dice is :

(A) $\frac{2}{3}$ (B) $\frac{1}{6}$ (C) $\frac{5}{6}$ (D) $\frac{5}{36}$

98. $\vec{i}\cdot(\vec{j}\times\vec{k})+\vec{j}\cdot(\vec{i}\times\vec{k})+\vec{k}\cdot(\vec{i}\times\vec{j})=$

(A) 0 (B) 1 (C) $\frac{\pi}{4}$ (D) 3

99. The two vectors $2\vec{i}+5\vec{j}+\vec{k}$ and $3\vec{i}+2\vec{j}+4\vec{k}$ are :

(A) parallel (B) perpendicular

(C) equal (D) none of these

100. $\int\frac{dx}{x-1}=$

(A) $\log|x+1|+k$ (B) $-\log|1+x|+k$

(C) $\log|x-1|+k$ (D) $\log x+k$

SECTION – B : Non-Objective Type Questions

SHORT ANSWER TYPE QUESTIONS

Direction : *Question Nos. 1 to 30 are of short answer type. Answer only 15 questions from these.* $15\times 2=30$

1. If $A=\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}$, then find AB.

2. Find the value of x and y if :

$$2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

3. Using property of determinants show that :

$$\begin{vmatrix}x+y & 2x & 2x\\ 2x & x+y & 2x\\ 2x & 2x & x+y\end{vmatrix}=(5x+y)(y-x)^2$$

4. Evaluate :

$$\Delta=\begin{vmatrix}1+a_1 & a_2 & a_3\\ a_1 & 1+a_2 & a_3\\ a_1 & a_2 & 1+a_3\end{vmatrix}$$

5. Differentiate : $\cos(\sin x)$

6. If $y=\sin(\cot x)$, then find $\frac{dy}{dx}$

7. If $xy=x^3+y^3$, find $\frac{dy}{dx}$

8. Evaluate : $\int x\log 2x\,dx$

9. Evaluate : $\int\sin 7x\sin x\,dx$

10. Integrate : $\int\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}\,dx$

11. Integrate : $\int\frac{\cos x}{\sqrt{1+\sin x}}\,dx.$

12. Integrate : $\int\sin^{-1}x\,dx$

13. Find $\int_0^{\pi/4}\tan^2 x\,dx$

14. Evaluate : $\int_0^{\pi/4}\sin 2x\sin 3x\,dx$

15. Evaluate : $\int_2^3\frac{1}{x\log x}\,dx$

16. Solve : $\sqrt{a+x}\frac{dy}{dx}+x=0$

17. Prove that the vectors $\vec{i}-2\vec{j}+5\vec{k}$ and $-2\vec{i}+4\vec{j}+2\vec{k}$ are mutually perpendicular.

18. If $\vec{a}+\vec{b}+\vec{c}=0$ then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$:

19. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ be coplanar, show that $c^2=ab$.

20. Find the equation of the line through the point $(-1, 2, 3)$ which is perpendicular to the lines $\frac{x}{2}=\frac{y-1}{-3}=\frac{z+2}{-2}$ and $\frac{x+3}{-1}=\frac{y+2}{2}=\frac{z-1}{3}$.

21. If $P(A)=0.4$, $P(B)$, $P(B)=p$, $P(A\cup B)=0.6$ and A, B are given to be independent events find the value of p.

22. Prove that : $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

23. Prove that :

$$2\tan^{-1}x=\sin^{-1}\frac{2x}{1+x^2}=\cos^{-1}\frac{1-x^2}{1+x^2}=\tan^{-1}\frac{2x}{1-x^2}$$

24. **Solve for x : $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$**

25. **Evaluate : $\int \frac{\log x}{x} dx$**

26. **Write the value of**

$$\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$$

27. **Differentiate : $2\sqrt{\cot(x^2)}$**

28. **Solve : $x\,dy + y\,dx = xy \cdot dy$**

29. **Find the area between the x-axis and the curve $y = \sin x$, from $x = 0$ to $x = \pi$.**

30. **Find $\vec{a} \times \vec{b}$, where $\vec{a} = 2\hat{i} - 3\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j}$.**

LONG ANSWER TYPE QUESTIONS

Direction : *Question Nos. 31 to 38 are long answer type question. Answer any 4 questions from these.* $5 \times 4 = 20$

31. **If $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, then show that**

$$\frac{dy}{dx} - \sec x = 0$$

32. **Find the area of the region bounded by $y^2 = 4x$, $x = 1$, $x = 4$ and x-axis in the first quadrant.**

33. **Prove that the points A, B, C with position vectors $\vec{a}, \vec{b}, \vec{c}$ are collinear if and only if**

$$(\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) + (\vec{a} \times \vec{b}) = \vec{0}$$

34. **Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. Also find the coordinates of the foot of the perpendicular and the equation of the perpendicular.**

35. **Find the probability of at most two tails or at least two heads in a toss of 3 coins.**

36. **Prove that :**

$$\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

37. **Minimize** $z = 2x + y$

subject to $x + y \geq 1$

$x + 2y \leq 10$

$y \leq 4$

$x \geq 0, y \geq 0$

38. **Evaluate :** $\int_0^{\pi/4} \log(1 + \tan x)\, dx$

ANSWER WITH EXPLANATIONS

SECTION – A

OMR ANSWER-SHEET

1.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	51.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
2.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	52.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
3.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	53.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
4.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	54.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
5.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	55.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
6.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	56.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
7.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	57.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
8.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	58.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
9.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	59.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
10.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	60.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
11.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	61.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
12.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	62.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
13.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	63.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
14.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	64.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
15.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	65.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
16.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	66.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
17.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	67.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
18.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	68.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
19.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	69.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
20.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	70.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
21.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	71.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
22.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	72.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
23.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	73.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
24.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	74.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
25.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	75.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
26.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	76.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
27.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	77.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
28.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	78.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
29.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	79.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
30.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	80.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
31.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	81.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
32.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	82.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
33.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	83.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
34.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	84.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
35.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	85.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
36.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	86.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
37.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	87.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
38.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	88.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
39.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	89.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
40.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	90.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
41.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	91.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
42.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	92.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
43.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	93.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
44.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	94.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
45.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	95.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
46.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	96.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
47.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	97.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
48.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	98.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
49.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	99.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
50.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	100.	Ⓐ	Ⓑ	Ⓒ	Ⓓ

ANSWER

1. (A)	2. (C)	3. (A)	4. (A)	5. (D)
6. (C)	7. (A)	8. (A)	9. (A)	10. (C)
11. (C)	12. (B)	13. (C)	14. (C)	15. (A)
16. (D)	17. (D)	18. (B)	19. (A)	20. (A)
21. (B)	22. (B)	23. (B)	24. (C)	25. (C)
26. (B)	27. (B)	28. (B)	29. (A)	30. (D)
31. (C)	32. (A)	33. (A)	34. (B)	35. (B)
36. (C)	37. (A)	38. (D)	39. (B)	40. (B)
41. (B)	42. (B)	43. (A)	44. (B)	45. (B)
46. (B)	47. (C)	48. (A)	49. (A)	50. (B)
51. (A)	52. (A)	53. (B)	54. (C)	55. (D)
56. (C)	57. (D)	58. (C)	59. (B)	60. (C)
61. (C)	62. (A)	63. (A)	64. (D)	65. (D)
66. (C)	67. (D)	68. (B)	69. (A)	70. (B)
71. (A)	72. (A)	73. (C)	74. (A)	75. (C)
76. (D)	77. (B)	78. (B)	79. (D)	80. (C)
81. (A)	82. (B)	83. (A)	84. (D)	85. (C)
86. (C)	87. (C)	88. (A)	89. (D)	90. (C)
91. (B)	92. (D)	93. (D)	94. (C)	95. (B)
96. (B)	97. (B)	98. (D)	99. (B)	100. (C)

SECTION – B

1. $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

$$AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5\times2+(-1)\times3 & 5\times1+(-1)\times4 \\ 6\times2+7\times3 & 6\times1+7\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

2. $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix}+\begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix}+\begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix}=\begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$\therefore\ 2 + y + 5 \quad \Rightarrow y = 3$

and $2x + 2 = 8 \quad \Rightarrow 2x = 6 \Rightarrow x = 3$

$\therefore\ x = 3, y = 3$

3. $\begin{vmatrix} x+y & 2x & 2x \\ 2x & x+y & 2x \\ 2x & 2x & x+y \end{vmatrix}$

Applying for, $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 5x+y & 2x & 2x \\ 5x+y & x+y & 2x \\ 5x+y & 2x & x+y \end{vmatrix}$$

$$= (5x+y)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+y & 2x \\ 1 & 2x & x+y \end{vmatrix}$$

Applying for, $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$

$$= (5x+y)\begin{vmatrix} 1 & x-y & 0 \\ 0 & x+y & 2x \\ 0 & x-y & -x+y \end{vmatrix}$$

$= (5x + y)(y - x)^2$; Proved.

4. $\Delta = \begin{vmatrix} 1+a_1 & a_2 & a_3 \\ a_1 & 1+a_2 & a_3 \\ a_1 & a_2 & 1+a_3 \end{vmatrix}$

Applying for $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 1+a_1+a_2+a_3 & a_2 & a_3 \\ 1+a_1+a_2+a_3 & 1+a_2 & a_3 \\ 1+a_1+a_2+a_3 & a_2 & 1+a_3 \end{vmatrix}$$

$$= (1+a_1+a_2+a_3)\begin{vmatrix} 1 & a_2 & a_3 \\ 1 & 1+a_2 & a_3 \\ 1 & a_2 & 1+a_3 \end{vmatrix}$$

Applying for $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$(1 + a_1 + a_2 + a_3)\begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & a_2 & 1+a_3 \end{vmatrix}$$

$= (1 + a_1 + a_2 + a_3)\ \{1\ (-1 \times -1 - 0 \times 1)\}$

$= 1 + a_1 + a_2 + a_3$

5. Let $y = \cos(\sin x)$

Now $\dfrac{dy}{dx} = \dfrac{d}{dx}\{\cos(\sin x)\}$

$$= \frac{d\{\cos(\sin x)\}}{d\sin x}\cdot\frac{d\sin x}{dx}$$

$= -\sin(\sin x)\cdot\cos x = -\cos x\sin(\sin x)$ **Ans.**

6. $\dfrac{dy}{dx} = \dfrac{d}{dx}\{\sin(\cot x)\}$

$$= \frac{d\{\sin(\cot x)\}}{d\cot x}\cdot\frac{d(\cot x)}{dx}$$

$= \cos(\cot x)\cdot(-\operatorname{cosec}^2 x)$

$= -\operatorname{cosec}^2 x\cos(\cot x)$ **Ans.**

7. Given, $xy = x^3 + y^3$

Differentiating w.r.t. to x, we get

$$\frac{d}{dx}(xy) = \frac{d}{dx}(x^3)+\frac{d}{dx}(y^3) \quad ...(1)$$

$$1\cdot y + x\cdot\frac{dy}{dx} = 3x^2 + 3y^2\frac{dy}{dx}$$

$\Rightarrow (x - 3y^2)\dfrac{dy}{dx} = 3x^2 - y \ \therefore\ \dfrac{dy}{dx} = \dfrac{3x^2-y}{x-3y^2}$ **Ans.**

8. Let, $I = \int x\log 2x\, dx$

$$= \log 2x\int x\,dx - \int\left[\frac{d}{dx}\log 2x\int x\,dx\right]dx + c$$

$$= \log 2x\,.\,\frac{x^2}{2} - \int\frac{1}{2x}.2.\frac{x^2}{2}dx + c$$

$$= \frac{x^2}{2}\log 2x - \frac{1}{2}\int x\, dx + c$$

$$= \frac{x^2}{2}\log 2x - \frac{1}{2}.\frac{x^2}{2} + c$$

$$= \frac{x^2}{2}\log 2x - \frac{x^2}{4} + c$$ **Ans.**

9. Let, I $= \int \sin 7x \sin x\, dx$

$$= \frac{1}{2}\int 2\sin 7x \sin x\, dx$$

$$= \frac{1}{2}\int [\cos(7x - x) - \cos(7x + x)]\, dx$$

$$= \frac{1}{2}\int (\cos 6x - \cos 8x)\, dx$$

$$= \frac{1}{2}\left[\frac{\sin 6x}{6} - \frac{\sin 8x}{8}\right] + c$$

$$= \frac{\sin 6x}{12} - \frac{\sin 8x}{16} + c$$ **Ans.**

10. Let, I $= \int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}}\, dx$

$$= \int \frac{\sin x + \cos x}{\sqrt{\sin^2 x + \cos^2 x + 2\sin x.\cos x}}\, dx$$

$$= \int \frac{\sin x + \cos x}{\sin x + \cos x}\, dx$$

$= \int dx = x + c;$ **Ans.**

11. $\int \frac{\cos x}{\sqrt{1 + \sin x}}\, dx = \int (1 + \sin x)^{-1/2} \cos x\, dx$

Let, $1 + \sin x = t \Rightarrow \cos x = \frac{dt}{dx} \Rightarrow dx = \frac{dt}{\cos x}$

$\therefore \int (1 + \sin x)^{-1/2} \cos x\, dx$

$$= \int (t)^{-1/2} \cos x \frac{dt}{\cos x} = \frac{t^{\frac{-1}{2}+1}}{\left(-\frac{1}{2}+1\right)} + C$$

$$= 2t^{1/2} + C = 2\sqrt{1 + \sin x} + C$$

12. Let, $I = \int \sin^{-1} x\, dx$

$= \int \sin^{-1} x\, .\, 1 dx$

$$= \sin^{-1} x \int dx - \int\left(\frac{d}{dx}\sin^{-1} x. \int 1 dx\right)dx$$

$$= \sin^{-1} x.x - \int \frac{1}{\sqrt{1 - x^2}} x\, dx$$

$$= x.\sin^{-1} x + \frac{1}{2}\int \frac{-2}{\sqrt{1 - x^2}} dx$$

$$= x.\sin^{-1} x + \frac{1}{2} \times 2\sqrt{1 - x^2} + C$$

$$= x.\sin^{-1} x + \sqrt{1 - x^2} + C$$ **Ans.**

13. $\int \tan^2 x\, dx = \int (\sec^2 x - 1)\, dx = \tan x - x$

$$\therefore I = \int_0^{\pi/4} \tan^2 x dx = [\tan x - x]_0^{\pi/4}$$

$$= \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (\tan 0 - 0) = 1 - \frac{\pi}{4}$$ **Ans.**

14. $\int_0^{\pi/4} \sin 2x \sin 3x\, dx$

$$= \frac{1}{2}\int_0^{\pi/4} (2\sin 3x \sin 2x)\, dx$$

$$= \frac{1}{2}\int_0^{\pi/4} (\cos x - \cos 5x)\, dx$$

$$= \frac{1}{2}\left[\sin x - \frac{\sin 5x}{5}\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left[\sin\frac{\pi}{4} - \frac{1}{5}\sin\frac{5\pi}{4} - 0\right]$$

$$= \frac{1}{2}\left[\sin\frac{\pi}{4} + \frac{1}{5}\sin\frac{\pi}{4}\right]$$

$$= \frac{1}{2}\left(1 + \frac{1}{5}\right)\frac{1}{\sqrt{2}} = \frac{3}{5\sqrt{2}}$$ **Ans.**

15. $\int_2^3 \frac{1}{x \log x} dx$

Putting $\log x = y$

$$\frac{1}{x} = \frac{dy}{dx} \Rightarrow dy = \frac{dx}{x}$$

$$= \int_{\log 2}^{\log 3} \frac{1}{y} dy = [\log y]_{\log 2}^{\log 3}$$

$$= \log(\log 3) - \log(\log 2) = \log\frac{\log 3}{\log 2}$$ **Ans.**

16. $\sqrt{a + x}\frac{dy}{dx} + x = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{a + x}}$$

$$\Rightarrow dy = \frac{-x}{\sqrt{a + x}} dx$$

$$\Rightarrow \int dy = -\int \frac{x}{\sqrt{a + x}} dx$$

$$\Rightarrow y = -\int \frac{a + x - a}{\sqrt{a + x}} dx$$

$$\Rightarrow y = -\int \sqrt{a + x} dx + a\int (a + x)^{-1/2} dx$$

$$\Rightarrow y = -\frac{2}{3}(a + x)^{3/2} + 2a\sqrt{a + x} + C$$

17. Let $\vec{A} = \hat{i} - 2\hat{j} + 5\hat{k}$

$\vec{B} = 2\hat{i} + 4\hat{j} + 2\hat{k}$

When two vector perpendicular to each other then

$\vec{A}.\vec{B} = 0$

$\vec{A}.\vec{B} = (\vec{i}-2\vec{j}+5\vec{k}).(-2\vec{i}+4\vec{j}+2\vec{k})$

$= 1 \times -2 + (-2) \times 4 + 5 \times 2$

$= -2 - 8 + 10 = -10 + 10 = 0$

So $\vec{A}$ and $\vec{B}$ are mutually perpendicular.

18. $\vec{a}+\vec{b}+\vec{c} = 0$

$\Rightarrow \quad \vec{a} = -(\vec{b}+\vec{c})$

$\Rightarrow \quad \vec{b} = -(\vec{a}+\vec{c})$

$\vec{a}\times\vec{b} = \vec{a}\times(-\vec{a}+-\vec{c})$

$= -\vec{a}\times\vec{a} - \vec{a}\times\vec{c} = 0 - \vec{a}\times\vec{c} = \vec{c}\times\vec{a}$ ... (i)

$(\vec{b}\times\vec{c}) = (-\vec{a}-\vec{c})\times\vec{c} = -\vec{a}\times\vec{c} - \vec{c}\times\vec{c}$

$= \vec{c}\times\vec{a} - 0 = \vec{c}\times\vec{a}$ ... (ii)

from (i) & (ii)

$\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

19. As the given vectors are coplanar, we get

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

Expand by R_2

$\Rightarrow -1\,(ab - c^2) - 1\,(ac - ac) = 0$

$\Rightarrow -1\,(ab - c^2) = 0$

$\Rightarrow c^2 = ab$

20. Let the line through (–1, 2, 3) which is to be ⊥ to the given lines have d.c's l, m, n.

Then, $l \times 2 + m \times (-3) + n \times (-2) = 0$

and $l \times (-1) + m \times 2 + n \times 3 = 0$

$\therefore \quad 2l - 3m - 2n = 0$ ... (1)

and $-l + 2m + 3n = 0$ ... (2)

From (1) and (2)

$$\frac{l}{-9+4} = \frac{m}{2-6} = \frac{n}{4-3}$$

$$\Rightarrow \frac{l}{-5} = \frac{m}{-4} = \frac{n}{1}$$

$$\Rightarrow \frac{l}{5} = \frac{m}{4} = \frac{n}{-1}$$

Hence direction ratios of the line are 5, 4, –1

∴ The required equation of the line is

$$\frac{x+1}{5} = \frac{y-2}{4} = \frac{z-3}{-1}$$

21. Since A and B are independent events

$\therefore \quad P(A \cap B) = P(A) \cdot P(B)$

We have, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

[By addition theorem]

$\Rightarrow \quad P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$\Rightarrow \quad 0.6 = 0.4 + p - 0.4p$

$\Rightarrow \quad 0.6\,p = 0.2$

$\therefore \quad p = \frac{1}{3}$ **Ans.**

22. Let, $\sin^{-1}x = \theta$ then $\sin\theta = x$

But $\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$

$\therefore \quad \cos\left(\frac{\pi}{2}-\theta\right) = x$

$\Rightarrow \quad \cos^{-1} x = \frac{\pi}{2} - \theta$

$\Rightarrow \quad \theta + \cos^{-1}x = \frac{\pi}{2}$

$\therefore \quad \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$. **Proved**

23. Let, $\tan^{-1}x = \theta$

$\therefore \quad \tan\theta = x$

Now $\quad \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2x}{1+x^2}$

$\Rightarrow \quad 2\theta = \sin^{-1}\frac{2x}{1+x^2}$

$\therefore \quad 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}$

Again, $\quad \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-x^2}{1+x^2}$ $\quad [\therefore \tan\theta = x]$

$\Rightarrow \quad 2\theta = \cos^{-1}\frac{1-x^2}{1+x^2}$

$\therefore \quad 2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}$

Again, $\quad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\Rightarrow \quad \tan 2\theta = \frac{2x}{1-x^2}$ $\quad [\because \tan\theta = x]$

$\Rightarrow \quad 2\theta = \tan^{-1}\frac{2x}{1-x^2}$

$\therefore \quad 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$

Hence, $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}$

$= \tan^{-1}\frac{2x}{1-x^2}$. **Proved.**

24. $\because \tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

$\therefore \quad \tan^{-1}\left(\frac{2x+3x}{1-(2x)(3x)}\right) = \frac{\pi}{4} \Rightarrow \frac{5x}{1-6x^2} = \tan\frac{\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2} = 1 \Rightarrow 5x = 1 - 6x^2 \Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow 6x^2 + 6x - x - 1 = 0 \Rightarrow 6x(x+1) - 1\,(x+1) = 0$

$\Rightarrow (x+1)(6x-1) = 0 \;\therefore x \neq -1$ and $x = \frac{1}{6}$ **Ans.**

25. Let $I = \int \frac{\log x}{x} dx$

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$\therefore \quad I = \int t\, dt = \frac{t^2}{2} + c = \frac{(\log x)^2}{2} + c$

26. Here, $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$

$= \frac{\pi}{3} - 2\left(-\frac{\pi}{6}\right) = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

27. Let $y = 2\sqrt{\cot(x^2)}$

Then $\frac{dy}{dx} = \frac{d(2\sqrt{\cot x^2})}{dx}$

$= 2\frac{d\sqrt{\cot x^2}}{dx}$

$= 2\frac{d\sqrt{\cot x^2}}{d\cot x^2} \cdot \frac{d\cot x^2}{dx^2} \cdot \frac{dx^2}{dx}$

$= 2 \cdot \frac{1}{2\sqrt{\cot x^2}} \cdot (-\text{cosec}^2 x^2) \cdot 2x$

$= -\frac{2x}{\sqrt{\cot x^2}} \text{cosec}^2 x^2$ **Ans.**

28. $x\,dy + y\,dx = xy \cdot dy$

divide by the dx on both sides,

$x\frac{dy}{dx} + y = xy$

$\Rightarrow \frac{dy}{y} + \frac{dx}{x} = x$

$\Rightarrow \log y + \log y = x + C$ **Ans.**

29. $y = \sin x$

Area $= \int_0^{\pi} \sin x\, dx = [-\cos x]_0^{\pi}$

$= -[\cos \pi - \cos 0°] = -[-1 - 1] = 2$ **Ans.**

30. Here $\vec{a} = 2\vec{i} - 3\vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{j}$.

$\therefore \quad \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -3 \\ 3 & 4 & 0 \end{vmatrix}$

$= \{0.0 - (-3).4\}\vec{i} + \{(-3).3 - 2.0)\vec{j} + (2.4 - 3.0)\vec{k}$

$= (0 + 12)\vec{i} + (-9 - 0)\vec{j} + (8 - 0)\vec{k}$

$= 12\vec{i} - 9\vec{j} + 8\vec{k}$ **Ans.**

31. $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$

$\frac{dy}{dx} = \frac{1}{\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)} \times \sec^2\left(\frac{\pi}{4} + \frac{x}{2}\right) \times \frac{1}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{1 + \tan^2\left(\frac{\pi}{4} + \frac{x}{2}\right)}{2\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\dfrac{2\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)}{1 + \tan^2\left(\frac{\pi}{4} + \frac{x}{2}\right)}}$

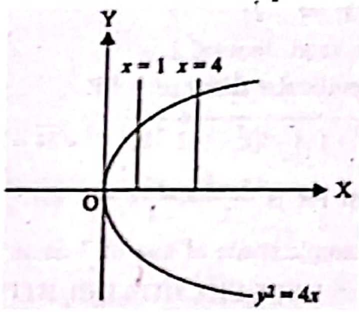
$\Rightarrow \frac{dy}{dx} = \frac{1}{\sin 2\left(\frac{\pi}{4} + \frac{x}{2}\right)}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sin\left(\frac{\pi}{2} + x\right)} = \frac{1}{\cos x} = \sec x$

$\Rightarrow \frac{dy}{dx} - \sec x = 0$

32. The reqd. area

= Area of the shaded portion



$= \int_1^4 y\,dx$

$= \int_1^4 \sqrt{4x}\,dx$

$= 2.\left[\frac{x^{3/2}}{3/2}\right]_1^4$

$= \frac{4}{3}[4^{3/2} - 1] = \frac{28}{3}$ sq. units

33. Given : $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$ and $\vec{OC} = \vec{c}$, where O is the origin.

$\vec{AB}$ = (Position vector of B) – (Position vector of A)

$= \vec{b} - \vec{a}$

and $\vec{BC}$ = (Position vector of C) – (Position vector of B)

$= \vec{c} - \vec{b}$

Now, A, B, C are collinear

$\Leftrightarrow$ $\vec{AB}$ and $\vec{BC}$ are parallel

$\Leftrightarrow (\vec{b}-\vec{a}) \times (\vec{c}-\vec{b}) = 0$

$\Leftrightarrow (\vec{b}-\vec{a}) \times \vec{c} - (\vec{b}-\vec{a}) \times \vec{b} = \vec{0}$ [By distributive law]

$\Leftrightarrow \vec{b} \times \vec{c} - \vec{a} \times \vec{c} - \vec{b} \times \vec{b} + \vec{a} \times \vec{b}) = \vec{0}$

[By distributive law]

$\Leftrightarrow (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) + (\vec{a} \times \vec{b}) = \vec{0}$

$[\because \vec{b} \times \vec{b} = \vec{0}$ and $-\vec{a} \times \vec{c} = \vec{c} \times \vec{a}]$

Thus, A, B, C are collinear

$\Leftrightarrow (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) + (\vec{a} \times \vec{b}) = \vec{0}$ **Proved.**

34. Any point on the given line,

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} \quad \text{... (1)}$$

is R $(2k + 1, -3k - 1, 8k - 10)$

If this is the foot of the $\perp$ from P (1, 0, 0) on (1), then

$(2k + 1 - 1).\ 2 + (-3k - 1 - 0).\ (-3) + (8k - 10 - 0).\ 8 = 0$

$\Rightarrow 4k + 9k + 3 + 64k - 80 = 0$

$\Rightarrow 77k = 77 \quad \Rightarrow \quad k = 1$

$\therefore$ R is $(3, -4, -2)$.

This is the reqd. foot of $\perp$.

Also perpendicular distance = PR

$= \sqrt{(3-1)^2 + (-4-0)^2 + (-2-0)^2} = \sqrt{24} = 2\sqrt{6}$

Also eq. of PR is $\frac{x-1}{2} = \frac{y}{-4} = \frac{z}{-2}$

35. The sample space of toss of 3 coins

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Let A = At most two tails

= {HHH, HHT, HTH, THH, HTT, THT, TTH}

B = At least two heads

= {HHT, HTH, THH, HHH}

$A \cap B$ = { HHT, HTH, THH, HHH} = B

Clearly n (A) = 7, n (B) = 4 = n (A $\cap$ B)

$\therefore$ P (A $\cup$ B) = P (A) + P (B) – P (A $\cap$ B)

= P (A) + P (B) – P (B)

= P (A) = $\frac{7}{8}$

36. Let, $\Delta = \begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$

$= (abc)\begin{vmatrix} a & c & c+a \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

[Taking a, b, c common from C_1, C_2 and C_3 respectively]

$= (abc)\begin{vmatrix} a & c & c+a \\ 0 & -2c & -2c \\ b & b+c & c \end{vmatrix}$ $[R_2 \to R_2 - (R_1 + R_3)]$

$= (abc)\begin{vmatrix} a & -a & c+a \\ 0 & 0 & -2c \\ b & b & c \end{vmatrix}$ $[C_2 \to C_2 - C_3]$

$= (abc)\ (2c) \cdot \begin{vmatrix} a & -a \\ b & b \end{vmatrix}$ [Expanding along R_2]

$= 2abc^2 \cdot (ab + ab) = 2abc^2\ (2ab) = 4a^2b^2c^2$ **Proved**

37. Here objective function is

Z = 2x + y

and constraints $x + y \geq 1$

$x + 2y \leq 10$

$y \leq 4$

$x \geq 0, y \geq 0$

$\because$ $x + y \geq 1$

Corresponding equation

$x + y = 1$

x	0	1
y	1	0

Check for (0. 0)

$\because$ $x + y \geq 1$

$0 + 0 \geq 1$

$0 \geq 1$ (false)

Shaded part is away from origin.

Again , $x + 2y \leq 10$

Corresponding equation

$x + 2y = 10$

x	0	10
y	5	0

Check for (0,0)

$\because$ $x + 2y \leq 10$

$0 + 0 \leq 10$

$0 \leq 10$ (True)

Shaded part is towards the origin.

$\because$ $x + 2y = 10$

$x + y = 1$

(–) (–) (–)

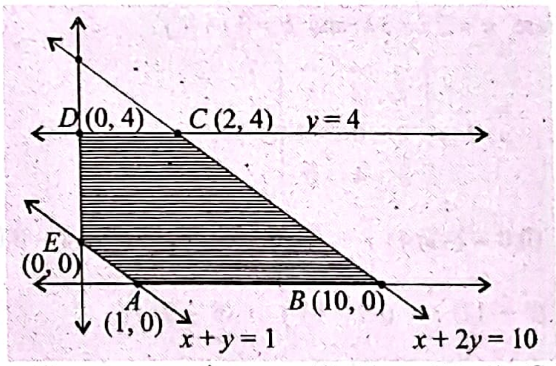
$y = 9$

$\because$ $x + y = 1$

$\Rightarrow x + 9 = 1$

$\therefore$ $x = -8$

$\therefore$ Point of intersection is (–8, 9) which is in second quadrant.



Here corner points are $A(1, 0)$, B (10, 0), C (2, 4), D (0, 4) and E (0, 1)

Corner point	$z = 2x + y$
A (1, 0)	$z = 2 \times 1 + 0 = 2$
B (10, 0)	$z = 2 \times 10 + 0 = 20$
C (2, 4)	$z = 2 \times 2 + 7 = 8$
D (0, 4)	$z = 2 \times 0 + 4 = 4$
E (0, 1)	$z = 2 \times 0 + 1 = 1$

$\therefore$ $z = 2x + y$ has minimum value '1' at the point E (0, 1)

Ans.

38. Let, $I = \int_0^{\pi/4} \log(1+\tan x)dx$ ...(A)

$$= \int_0^{\pi/4} \log\left\{1+\tan\left(\frac{\pi}{4}-x\right)\right\}dx$$

$$\left[\because \int_b^a f(x)dx = \int_b^a f(a-x)dx\right]$$

$$= \int_0^{\pi/4} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$$

$$= \int_0^{\pi/4} \log\left\{1+\frac{1-\tan x}{1+\tan x}\right\}dx$$

$$= \int_0^{\pi/4} \log\left(\frac{1+\tan x+1-\tan x}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log 2\, dx - \int_0^{\pi/4} \log(1+\tan x)dx$$

$$= \int_0^{\pi/4} \log 2\, dx - I \quad \text{[Using eq. (A)]}$$

$$\Rightarrow 2I = \log 2 \int_0^{\pi/4} dx \qquad \Rightarrow 2I = \log 2[x]_0^{\pi/4}$$

$$\Rightarrow 2I = \log 2\left[\frac{\pi}{4}-0\right] \qquad \Rightarrow 2I = \frac{\pi}{4}\log 2$$

$\therefore$ $I = \frac{\pi}{8}\log 2$ **Ans.**

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