MODEL PAPER – 2

Time : 3 Hours + 15 Minutes] [Total Marks : 100

Instructions to the Candidates :

1. Candidates are required to give their answers in their own words as far as practicable.
2. Figures in the right hand margin indicate full marks.
3. 15 Minutes of extra time has been allotted for the candidates to read the questions carefully.
4. This question paper is divided into two sections—**SECTION – A** and **SECTION – B**.
5. In **SECTION – A** there are **100 Objective Type Question,** out of which only 50 objective question be answered. Darken the circle with blue/black ball pen against the correct option on OMR Sheet provided to you. Do not use **Whitener/Liquid/Blade/Nail** on OMR paper; otherwise the result will be invalid.
6. In **SECTION – B**, there are **30 Short Answer Type Questions** (each carrying 2 marks), out of which any 15 questions are be answered.

   Apart from this, there are **8 Long Answer Type Question** (Each Carrying 5 marks), out of which 4 Questions are to be answered.
7. Use of any electronic device is prohibited.

SECTION – A : Objective Type Questions

Direction : *There are 100 Objective Type Questions, out of which only 50 objective questions to be answered. Mark the correct option on the OMR Answer Sheet.* $50 \times 1 = 50$

1. If the operation 'o' is defined as $aob = 3a + b$ then $(2 \text{ o } 3) \text{ o } 5 =$

   (A) 28 (B) 32 (C) 36 (D) 22

2. $|x| \ge 1, \tan\left[\frac{2}{3}\left(\tan^{-1}x + \cot^{-1}x\right)\right] =$

   (A) $\frac{1}{\sqrt{3}}$ (B) $\sqrt{3}$ (C) 0 (D) 1

3. $3\vec{k}.(13\vec{i} - 7\vec{k}) =$

   (A) 39 (B) 0 (C) –21 (D) 18

4. $(3\hat{k} - 7\hat{i}) \times 2\hat{k} =$

   (A) $-14\hat{j}$ (B) $14\hat{j}$

   (C) $11\hat{i} - 2\hat{k}$ (D) $2\hat{k} - 11\hat{i}$

5. The integrating factor of the differential equation $\frac{dy}{dx} - y\sin x = \cot x$ is :

   (A) $\sin x$ (B) $e^{-\sin x}$

   (C) $e^{\sin x}$ (D) $e^{\cos x}$

6. $P(A) = \frac{7}{11}, P(B) = \frac{9}{11}, P(A \cap B) = \frac{4}{11} \Rightarrow P(A/B) =$

   (A) $\frac{7}{9}$ (B) $\frac{4}{9}$ (C) 1 (D) $\frac{13}{22}$

7. If the direction ratios of two parallel lines are x, 5, 3 and 20, 10, 6 then the value of x is :

   (A) 10 (B) 5 (C) 3 (D) 40

8. $\int (\cos\theta\,\text{cosec}^2\theta - \cos\theta\cot^2\theta)\,d\theta =$

   (A) $\log \text{cosec}\,\theta + \cot\theta + k$ (B) $\text{cosec}\,\theta \cot\theta + k$

   (C) $k + \sin\theta$ (D) $\theta + k$

9. $\int (4\cos x - 5\sin x)\,dx =$

   (A) $k + 4\sin x + 5\cos x$ (B) $k - 4\sin x - 5\cos x$

   (C) $k + 4\sin x - 5\cos x$ (D) $k - 4\sin x + 5\cos x$

10. $\frac{d}{dx}\left(\frac{1}{3x-2}\right) =$

    (A) $\frac{-1}{(3x-2)^2}$ (B) $\frac{-3}{(3x-2)^2}$

    (C) $\frac{3}{(3x-2)^2}$ (D) $\frac{3}{3x-2}$

11. If $x = a\cos^2\theta, y = b\sin^2\theta$ then the value of $\frac{dy}{dx}$ is :

    (A) $\frac{b}{a}$ (B) $-\frac{b}{a}$

    (C) $\frac{b}{a}\sin 2\theta$ (D) $\frac{-b}{a}\tan^2\theta$

12. $\begin{vmatrix} 1 & 2 & -1 \\ 5 & 4 & 1 \\ 7 & 6 & 1 \end{vmatrix} =$

    (A) 0 (B) 1 (C) –1 (D) 12

13. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix} =$

    (A) $\begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & -3 \\ 0 & 5 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

14. Number of binary operations $\{a, b\}$ are :

(A) 8 (B) 4 (C) 16 (D) 64

15. If w is a non-real root of the equation $x^3 - 1 = 0$, then

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} =$$

(A) 0 (B) 1 (C) ω (D) ω^2

16. If $A = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ than $\Delta =$

(A) abc (B) 0

(C) a + b + c (D) None of these

17. $\frac{d}{dx}(\tan^{-1} x + \cos^{-1} x) =$

(A) $\frac{2}{1+x^2}$ (B) 0 (C) 1 (D) 2

18. If $y = \cos(\log x)$, then $\frac{dy}{dx} =$

(A) $-\sin(\log x)$ (B) $\frac{-\sin(\log x)}{x}$

(C) $\frac{\cos(\log x)}{x}$ (D) $-\sin(\log x)\log x$

19. $\int \frac{1-\cos 2x}{1+\cos 2x} dx =$

(A) $\tan x + x + k$ (B) $\tan x - x + k$

(C) $x - \tan^2 x + k$ (D) $\tan\frac{x}{2} + k$

20. $\int \frac{\sec x}{\sec x + \tan x} dx =$

(A) $\tan x + \sec x + k$ (B) $\tan x - \sec x + k$

(C) $\sec x + k$ (D) $\tan x + k$

21. The position vector of the point (x, y, z) is :

(A) $x\vec{i} - y\vec{j} - z\vec{k}$ (B) $x\vec{i} + y\vec{j} - z\vec{k}$

(C) $x\vec{i} + y\vec{j} + z\vec{k}$ (D) $x\vec{i} + y\vec{j} + z\vec{k}$

22. $|-\vec{i} + 2\vec{j} - 3\vec{k}| =$

(A) $\sqrt{15}$ (B) $\sqrt{3}$ (C) 2 (D) $\sqrt{14}$

23. Let $A = \{5, 6\}$, how many binary operations can be defined on this set ?

(A) 8 (B) 10 (C) 16 (D) 20

24. If $f : A \to B$ is one-one onto functions then :

(A) $n(A) > n(B)$ (B) $n(A) < (B)$

(C) $n(A) = n(B)$ (D) None of these

25. $\sin^{-1}\frac{1}{x} = ?$

(A) $\sec^{-1} x$ (B) $\mathrm{cosec}^{-1} x$

(C) $\tan^{-1} x$ (D) $\sin x$

26. If $\vec{a} = 2\vec{i} - 3\vec{j} + 4\vec{k}$ and $\vec{b} = \vec{i} + 2\vec{j} + \vec{k}$ then $\vec{a} + \vec{b} =$

(A) $\vec{i} + \vec{j} + 3\vec{k}$ (B) $3\vec{i} - \vec{j} + 5\vec{k}$

(C) $\vec{i} - \vec{j} - 3\vec{k}$ (D) $2\vec{i} + \vec{j} + \vec{k}$

27. If $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{b} = 3\vec{i} + 2\vec{j} + \vec{k}$, then $\cos\theta =$

(A) $\frac{6}{7}$ (B) $\frac{5}{7}$ (C) $\frac{4}{7}$ (D) $\frac{1}{2}$

28. The integrating factors of the linear differnetiation equation $\frac{dy}{dx} + y\tan x = \sec x$,

(A) sec x (B) log sec x

(C) sec x·tan$x$ (D) tan x

29. $\int \sin x \, dx =$

(A) $-\cos x + K$ (B) $\sin x + K$

(C) $-\sin x + K$ (D) $\tan x + K$

30. $\frac{d}{dx}(\cos x) =$

(A) $\cos x$ (B) $\sin x$

(C) $-\cos x$ (D) $-\sin x$

31. $\frac{d}{d\theta}(2\sin^2\theta + 2\cos^2\theta) =$

(A) 0 (B) 2

(C) $2\cos\theta - 2\sin\theta$ (D) $4\sin\theta\cos\theta$

32. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ then $A + A' = I_2$ if the value of x is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) π (D) 0

33. The value of x when $\begin{bmatrix} x & 15 \\ 4 & 4 \end{bmatrix} = 0$ is

(A) 15 (B) −15 (C) 4 (D) 4x

34. If $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{5}$, $P(A \cup B) = \frac{3}{4}$ then $P\left(\frac{A}{B}\right) =$

(A) $\frac{5}{12}$ (B) $\frac{5}{8}$ (C) $\frac{5}{4}$ (D) $\frac{5}{7}$

35. If A' and B' are independent events then :

(A) $P(A'B') = P(A).P(B)$ (B) $P(A'B') = P(A') + P(B')$

(C) $P(A'B') = P(A').P(B')$ (D) $P(A'B') = P(A') - P(B')$

36. If a line makes angle α, β and γ with the positive directions of x, y and z axes respectively, then :

(A) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 1 = 0$

(B) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$

(C) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 2$

(D) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

37. The distance between $(-4, -3, 7)$ and $(-1, 1, -5)$ is :

(A) 12 (B) 13

(C) 14 (D) None of these

38. $\int \frac{\sec^2(\log x)}{x} dx = ?$

(A) $\tan(\log x) + k$ (B) $-\tan(\log x) + k$

(C) $\cot(\log x) + k$ (D) $-\cot(\log x) + k$

39. $\int\sqrt{1-\sin 2x}\,dx = ?$

(A) $\sin x + \cos x + k$ (B) $\sin x - \cos x + k$
(C) $\cos x - \sin x + k$ (D) $\tan x - \cot x + k$

40. The value of determinant $\begin{vmatrix} 1 & 2 & 4 \\ 1 & 3 & 6 \\ 1 & 4 & 9 \end{vmatrix}$ is :

(A) 1 (B) −1 (C) 0 (D) 2

41. The direction cosines of x-axis are :

(A) 0, 0, 0 (B) 1, 0, 0
(C) 0, 1, 0 (D) 0, 0, 1

42. If $\vec{a} \perp \vec{b}$, then $\vec{a} \cdot \vec{b} =$

(A) 1 (B) −1
(C) 0 (D) None of these

43. $\vec{j}\cdot(\vec{k}\times\vec{i}) =$

(A) 0 (B) 1 (C) −1 (D) $\vec{j}$

44. Integration factor of differential equation $\frac{dy}{dx} + py = Q$, where P and IQ are functions of x is :

(A) $\int e^P dx$ (B) $e^{\int P dx}$
(C) $e^{-\int P dx}$ (D) None of these

45. $\frac{d}{dx}(2e^{2x}) =$

(A) $2e^{2x}$ (B) e^{2x} (C) $4e^{2x}$ (D) $2e^{x}$

46. $\frac{d}{dx}(e^{x-a}) =$

(A) e^{x-a} (B) $(x-a)\,e^{x-a}$
(C) e^{x} (D) $-e^{x-a}$

47. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) =$

(A) π (B) $-\frac{\pi}{3}, 0$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

48. $\sin\left(\sec^{-1}x + \text{cosec}^{-1}x\right) =$

(A) 1 (B) −1 (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{3}$

49. $\int_0^{\frac{\pi}{2}} \sin^3 x \cos x\, dx = ?$

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{\pi}{2}$

50. $\int_0^{\frac{\pi}{2}} \cos x\, dx = ?$

(A) −1 (B) 1 (C) $\frac{\pi}{2}$ (D) 0

51. $j \times k =$

(A) $\vec{i}$ (B) $-\vec{i}$ (C) $\vec{0}$ (D) 1

52. $\vec{a}\cdot\vec{b} =$

(A) $-\vec{b}\cdot\vec{a}$ (B) $\vec{b}\cdot\vec{a}$
(C) 1 (D) −1

53. $\int \frac{dx}{1+\cos x} =$

(A) $\tan\frac{x}{2} + k$ (B) $\frac{1}{2}\tan\frac{x}{2} + k$
(C) $2\tan\frac{x}{2} + k$ (D) $\tan^2\frac{x}{2} + k$

54. $\int_a^b x^5 dx =$

(A) $b^5 - a^5$ (B) $\frac{b^6 - a^6}{6}$ (C) $\frac{a^6 - b^6}{6}$ (D) $a^5 - b^5$

55. $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 6 & 6 \end{vmatrix} = \ldots\ldots$

(A) 0 (B) 12 (C) 24 (D) 5

56. The maximum value of $f(x) = \frac{\log x}{x}$ is :

(A) 1 (B) $\frac{2}{e}$ (C) e (D) $\frac{1}{e}$

57. If $f(x_1) = f(x_2) \Rightarrow x_1 = x_2\ \forall\ x_1, x_2 \in A$ then the function $f : A \to B$ is :

(A) one-one (B) one-one onto
(C) onto (D) many one

58. What type of a relation is $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ on the set $A = \{1, 2, 3, 4\}$.

(A) Reflexive (B) Transitive
(C) Symmetric (D) None of these

59. For any unit matrix I

(A) $I^2 = I$ (B) $|I| = 0$
(C) $|I| = 2$ (D) $|I| = 5$

60. If A, B and C are three independent events then :

(A) $P(ABC) = P(A) + P(B) + P(C)$
(B) $P(ABC) = P(A) - P(B) - P(C)$
(C) $P(ABC) = P(A) \cdot P(B) \cdot P(C)$
(D) None of these

61. If A and B be two events then $P\left(\frac{A}{B}\right) + P\left(\frac{A'}{B}\right) =$

(A) 0 (B) 1
(C) −1 (D) None of these

62. $\int e^{\log x}\, dx =$

(A) $\log x + k$ (B) $xe^{\log x} + k$
(C) $\frac{x^2}{2} + k$ (D) $\frac{\log x}{x} + k$

63. $\int \sin\theta \cdot \text{cosec}\,\theta\, d\theta =$

(A) $\sin\theta + k$ (B) $\text{cosec}\,\theta + k$
(C) $\frac{\sin^2\theta}{2} + k$ (D) $\theta + k$

64. If $y = \tan^{-1}\left[\frac{\sin x + \cos x}{\cos x - \sin x}\right]$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{\pi}{4}$ (C) 0 (D) 1

65. $\frac{d}{dx}[\tan x] = ?$

(A) $\sec^2 x$ (B) $\sec x$
(C) $\cot x$ (D) $-\sec^2 x$

66. $\tan^{-1}\frac{2x}{1-x^2}=$

(A) $2\sin^{-1}x$ (B) $\sin^{-1}2x$
(C) $\tan^{-1}2x$ (D) $2\tan^{-1}x$

67. $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{4}=.....$

(A) $\tan^{-1}\frac{3}{2}$ (B) $\tan^{-1}\frac{6}{7}$ (C) $\tan^{-1}\frac{5}{6}$ (D) $\tan^{-1}\frac{1}{2}$

68. Let A be a non-singular matrix of the order 2×2 then $|\text{adj } A|=$

(A) $2|A|$ (B) $|A|$ (C) $|A|^2$ (D) $|A|^3$

69. Intergrating factor of the differential equation $\frac{dy}{dx}+y\sec x=\tan x$ is :

(A) $\sec x+\tan x$ (B) $\sec x-\tan x$
(C) $\sec x$ (D) $\tan x\sec x$

70. If events A and B are mutually exclusive then :

(A) $P(A\cap B)=P(A).P(B)$ (B) $P(A\cap B)=0$
(C) $P(A\cap B)=1$ (D) $P(A\cup B)=0$

71. If A and B are any two independent events, then $P(AB')=$

(A) $P(A)P(B')$ (B) $P(A)+P(B')$
(C) $\frac{P(A)}{P(B')}$ (D) None of these

72. The angle between the straight lines $2x=3y=-z$ and $6x=-y=-4z$ is :

(A) $\frac{\pi}{2}$ (B) 0 (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

73. The distance of the point $(-3,-4,-5)$ from the origin is :

(A) 6 (B) $5\sqrt{2}$
(C) 50 (D) None of these

74. $|\vec{j}|=$

(A) 0 (B) 1 (C) 2 (D) 3

75. If $\vec{a}=2\vec{i}+\vec{j}+3\vec{k}$ and $\vec{b}=3\vec{i}+5\vec{j}-2\vec{k}$, then $|\vec{a}\times\vec{b}|=?$

(A) $\sqrt{307}$ (B) $\sqrt{407}$ (C) $\sqrt{207}$ (D) $\sqrt{507}$

76. $\int_0^{\pi/4}\tan^2\theta\,d\theta=$

(A) $1-\frac{\pi}{4}$ (B) $1+\frac{\pi}{4}$ (C) $-1-\frac{\pi}{4}$ (D) $\frac{\pi}{4}$

77. $\int_0^{\pi/4}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx=$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{-\pi}{2}$ (D) $\frac{-\pi}{4}$

78. If $\lambda\in R$ and $\Delta=\begin{vmatrix}a & b\\ c & d\end{vmatrix}$ then $\lambda\Delta=$

(A) $\begin{vmatrix}\lambda a & \lambda b\\ \lambda c & \lambda d\end{vmatrix}$ (B) $\begin{vmatrix}\lambda a & b\\ c & d\end{vmatrix}$

(C) $\begin{vmatrix}\lambda a & b\\ \lambda c & d\end{vmatrix}$ (D) None of these

79. If a, b, c are in A.P. then $-\begin{vmatrix}x+1 & x+2 & x+a\\ x+2 & x+3 & x+b\\ x+3 & x+4 & x+c\end{vmatrix}=$

(A) 3 (B) -3 (C) 0 (D) 1

80. If $y+x=\sin(y+x)$, then $\frac{dy}{dx}=$

(A) $\frac{1-\cos(y+x)}{1+\cos(y+x)}$ (B) 1
(C) -1 (D) 0

81. $\frac{d}{dx}(e^{3x})=$

(A) e^{3x} (B) $\frac{e^{3x}}{3}$ (C) $3e^{3x}$ (D) $3e^{x}$

82. $A=\begin{bmatrix}1 & 2\\ -1 & -3\end{bmatrix}\Rightarrow A'=$

(A) $\begin{bmatrix}-1 & -2\\ 1 & 3\end{bmatrix}$ (B) $\begin{bmatrix}1 & -1\\ 2 & -3\end{bmatrix}$ (C) $\begin{bmatrix}1 & 1\\ 2 & 3\end{bmatrix}$ (D) $\begin{bmatrix}-1 & 1\\ 2 & 3\end{bmatrix}$

83. If $A=[1, 2]$, $B=\begin{bmatrix}1\\ 2\end{bmatrix}$, then $AB=$

(A) $[5]$ (B) $\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$
(C) $[1\quad 4]$ (D) None of these

84. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

85. Solve for x : $\sin^{-1}2x+\sin^{-1}3x=\frac{\pi}{3}$

(A) $\sqrt{\frac{76}{3}}$ (B) $\sqrt{\frac{3}{76}}$ (C) $\frac{3}{\sqrt{76}}$ (D) $\frac{\sqrt{3}}{76}$

86. $\frac{d}{dx}\left[\lim_{x\to a}\frac{x^5-a^5}{x-a}\right]=$

(A) $5a^4$ (B) $5x^4$ (C) 1 (D) 0

87. A line passing through $(2,-1,3)$ and its direction ratios are $3,-1,2$. The equation of the line is :

(A) $\frac{x+2}{3}=\frac{y-1}{-1}=\frac{z+3}{2}$ (B) $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$

(C) $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z-2}{3}$ (D) $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z-2}{3}$

88. $\int\sqrt{1+\cos 2x}\,dx=$

(A) $\sqrt{2}\cos x+C$ (B) $\sqrt{2}\sin x+C$
(C) $-\cos x-\sin x+C$ (D) $\sqrt{2}\sin\frac{x}{2}+C$

89. If the position vectors of the point A and B be repsectively (1, 2, 3) and (–3, –4, 0) then $\overrightarrow{AB}=$

(A) $4\vec{i}+6\vec{j}+3\vec{k}$ (B) $-4\vec{a}-6\vec{j}-3\vec{k}$

(C) $-3\vec{i}-8\vec{k}$ (D) $-3\vec{i}+8\vec{j}$

90. If $\vec{a}=3\vec{i}+2\vec{j}+\vec{k}, \vec{b}=4\vec{i}-5\vec{j}+3\vec{k}$, then $\vec{a}\cdot\vec{b}=$

(A) 2 (B) 3 (C) 5 (D) 7

91. The order of the differential equation $\frac{d^2y}{dx^2}+x^3\left(\frac{dy}{dx}\right)^3=x^4$ is :

(A) 1 (B) 2 (C) 4 (D) 3

92. The differential co-efficient of log $\sqrt{x}$ w.r.t. x

(A) $\frac{1}{2x}$ (B) $\frac{1}{2\sqrt{x}}$ (C) $\frac{\sqrt{x}}{2}$ (D) $\sqrt{x}$

93. If $y=x^5$ then $\frac{dy}{dx}=$ ....

(A) $5x$ (B) $6x$

(C) $5x^4$ (D) $5x^2$

94. If $y=\sin^2 x$, then $\frac{dy}{dx}$

(A) $2\sin x$ (B) $\cos^2 x$

(C) $2\sin x\cos x$ (D) $\sin x\cdot\cos x$

95. If $\begin{vmatrix}2 & 4\\5 & 1\end{vmatrix}=\begin{vmatrix}2x & 4\\6 & x\end{vmatrix}$ Then the value of x is :

(A) ± 2 (B) $\pm\frac{1}{3}$ (C) $\pm\sqrt{3}$ (D) $\pm(0.5)$

96. If $A=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, then :

(A) A^{-1} exists (B) $|A|=0$

(C) A^{-1} does not exist (D) None of these

97. If A and B are events such that $P(A\cup B)=\frac{3}{4}$, $P(A\cap B)=\frac{1}{4}$, $P(A)=\frac{2}{3}$ then P (AB) is :

(A) $\frac{3}{8}$ (B) $\frac{5}{8}$ (C) $\frac{5}{12}$ (D) $\frac{1}{4}$

98. If one card is drawn out of 52 playing cards, the probability that it is an ace is :

(A) $\frac{1}{26}$ (B) $\frac{1}{13}$ (C) $\frac{1}{52}$ (D) $\frac{1}{4}$

99. If $x\vec{i}-3\vec{j}+5\vec{k}$ and $-x\vec{i}+x\vec{j}+2\vec{k}$ are perpendicular to each other then the value of $x=$

(A) –2, 5 (B) 2, 5

(C) –2, –5 (D) 2, –5

100. $\vec{i}\times(\vec{i}\times\vec{j})+\vec{j}\times(\vec{j}\times\vec{k})+\vec{k}\times(\vec{k}\times\vec{i})=$

(A) $\vec{i}+\vec{j}+\vec{k}$ (B) 0

(C) 1 (D) $-(\vec{i}+\vec{j}+\vec{k})$

SECTiON – B : Non-Objective Type Questions

SHORT ANSWER TYPE QUESTIONS

Direction : *Question Nos. 1 to 30 are of short answer type. Answer only 15 questions from these.* $15\times 2=30$

1. **Evaluate :** $\int\cos 2x\cos 4x\,dx$

2. **Integrate :** $\int\sqrt{1+\cos 2x}\,dx$

3. **Evaluate :** $\int\frac{\log x}{x}dx$

4. **Integrate :** $\int\frac{x+2}{x+4}dx$

5. **Evaluate :** $\int\frac{\sin^2 x}{1+\cos x}dx$

6. **Evaluate** $\int_0^{\pi/2}\cos^3 x\,dx$

7. **Evaluate :** $\int_0^{\pi/4}(\tan x-x)\tan^2 x\,dx$

8. **Find the value of** $\int_0^{2\pi}|\sin x|\,dx$

9. **If the function $f: R\to R$, defined by $f(x)=3x-4$, is invertible, find f^{-1}.**

10. **If** $A=\begin{bmatrix}3 & -5\\-4 & 2\end{bmatrix}$, **then verify that** $A^2-5A-14I=0$

11. **If** $A=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}$ **and** $B=\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}$, **then find A – B.**

12. **Using properties of determinants, prove the following :**

$$\begin{vmatrix}x & y & z\\x^2 & y^2 & z^2\\x^3 & y^3 & z^3\end{vmatrix}=xyz(x-y)(y-z)(z-x)$$

13. **Differentiate :** $\cos\sqrt{x}$

14. **If** $y=\sin\sqrt{\cos x}$, **find** $\frac{dy}{dx}$

15. **If** $x+y=\sin(xy)$, **find** $\frac{dy}{dx}$

16. **Find the particular solution of the differential equation** $\frac{dy}{dx}-\frac{y}{x}+\operatorname{cosec}\left(\frac{y}{x}\right)=0$, $y=0$, when $x=1$

17. **Solve the differential equation** $(x^3+y^3)\,dy-x^2y\,dx=0$.

18. **Find the value of p so that the vectors** $\hat{i}+2\hat{j}-3\hat{k}$, $p\hat{i}-\hat{j}+\hat{k}$ **and** $3\hat{i}-4\hat{j}+5\hat{k}$ **are coplanar.**

19. **If** $|\vec{a}|=\sqrt{3}, |\vec{b}|=2$ **and angle between** $\vec{a}$ **and** $\vec{b}$ **is 60°, find** $\vec{a}.\vec{b}$

20. Show that the lines $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5}$ and $\frac{x-2}{4}=\frac{y-1}{4}=\frac{z+1}{-2}$ do not intersect each other.

21. Two dice are thrown. Find the probability of getting an odd number of the first and a multiple of 3 on other.

22. Prove that : $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

23. Prove : $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left|xy-\sqrt{(1-x^2)(1-y^2)}\right|$

24. Prove that $4\,(\cot^{-1} 3 \pm \operatorname{cosec}^{-1}\sqrt{5}) = \pi$

25. Write the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$.

26. Differentiate : $\sin[\cos x^2)]$

27. Show that the vectors (5, –4, 2) and (2, 1, –3) are perpendicular to one another.

28. If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, then find $P\left(\frac{A'}{B'}\right)$ and $P\left(\frac{B'}{A'}\right)$

29. Find $\frac{dy}{dx}$, when $y = \frac{\sin^2 x}{\sqrt{\cos x}}$

30. Find $\frac{dy}{dx}$, when $y = \sin(\log x)$.

LONG ANSWER TYPE QUESTIONS

Direction : *Question Nos. 31 to 38 are long answer type question. Answer any 4 questions from these.* $5 \times 4 = 20$

31. If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$

32. Investigate the values of x for which the function $x^5 - 5x^4 + 5x^3 - 1$ has maximum or minimum or neither. Find also its maximum and minimum values.

33. Prove by vector method that the area of $\Delta\, ABC$ is $\frac{a^2 \sin B \sin C}{2 \sin A}$

34. Write the direction cosines of a line parallel to the line $\frac{3-x}{3}=\frac{y+2}{-2}=\frac{z+2}{6}$

35. Two coins are tossed. What is the probability of coming up of two heads, if it is know that at least one head come up?

36. Factorize $\begin{bmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{bmatrix}$

37. Solve the following linear programming problem :
Maximize $z = 2x + 3y$
Subject to the constraints

$$3x + 4y \le 12$$
$$x \ge 0,\ y \ge 0.$$

38. Prove that $\int_0^{\pi/2} \log \tan x\, dx = 0$

ANSWER WITH EXPLANATIONS

SECTION – A

OMR ANSWER-SHEET

1.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	51.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
2.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	52.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
3.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	53.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
4.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	54.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
5.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	55.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
6.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	56.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
7.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	57.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
8.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	58.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
9.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	59.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
10.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	60.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
11.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	61.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
12.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	62.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
13.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	63.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
14.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	64.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
15.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	65.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
16.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	66.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
17.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	67.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
18.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	68.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
19.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	69.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
20.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	70.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
21.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	71.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
22.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	72.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
23.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	73.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
24.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	74.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
25.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	75.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
26.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	76.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
27.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	77.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
28.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	78.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
29.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	79.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
30.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	80.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
31.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	81.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
32.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	82.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
33.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	83.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
34.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	84.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
35.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	85.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
36.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	86.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
37.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	87.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
38.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	88.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
39.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	89.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
40.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	90.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
41.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	91.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
42.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	92.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
43.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	93.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
44.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	94.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
45.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	95.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
46.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	96.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
47.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	97.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
48.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	98.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
49.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	99.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
50.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	100.	Ⓐ	Ⓑ	Ⓒ	Ⓓ

ANSWER

1. (B)	2. (B)	3. (C)	4. (B)	5. (D)
6. (B)	7. (A)	8. (C)	9. (A)	10. (B)
11. (B)	12. (A)	13. (A)	14. (C)	15. (A)
16. (B)	17. (B)	18. (B)	19. (B)	20. (B)
21. (D)	22. (D)	23. (C)	24. (C)	25. (B)
26. (B)	27. (B)	28. (A)	29. (A)	30. (D)
31. (A)	32. (B)	33. (A)	34. (A)	35. (C)
36. (D)	37. (B)	38. (A)	39. (A)	40. (A)
41. (B)	42. (C)	43. (B)	44. (B)	45. (C)
46. (A)	47. (B)	48. (A)	49. (B)	50. (B)
51. (A)	52. (B)	53. (A)	54. (B)	55. (C)
56. (D)	57. (A)	58. (D)	59. (A)	60. (C)
61. (B)	62. (C)	63. (D)	64. (D)	65. (A)
66. (D)	67. (B)	68. (B)	69. (A)	70. (B)
71. (A)	72. (A)	73. (B)	74. (B)	75. (B)
76. (A)	77. (A)	78. (C)	79. (C)	80. (C)
81. (C)	82. (B)	83. (A)	84. (D)	85. (B)
86. (D)	87. (B)	88. (B)	89. (B)	90. (C)
91. (B)	92. (A)	93. (C)	94. (C)	95. (C)
96. (A)	97. (B)	98. (B)	99. (D)	100. (D)

SECTION – B

1. Let, $I = \int \cos 2x \cos 4x \, dx$

$$= \frac{1}{2}\int 2\cos 2x \cos 4x \, dx$$

$$= \frac{1}{2}\int [\cos(2x+4x) + \cos(2x-4x)]\, dx$$

$$= \frac{1}{2}\int (\cos 6x + \cos 2x)\, dx$$

$$= \frac{1}{2}\left[\frac{\sin 6x}{6} + \frac{\sin 2x}{2}\right] + c$$

$$= \frac{\sin 6x}{12} + \frac{\sin 2x}{4} + c \quad \textbf{Ans.}$$

2. Let, $I = \int \sqrt{1+\cos 2x}\, dx = \int \sqrt{2\cos^2 x}\, dx$

$$= \sqrt{2}\int \cos x \, dx = \sqrt{2}\sin x + c; \quad \textbf{Ans.}$$

3. Let, $I = \int \frac{\log x}{x} dx$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$$\therefore \quad I = \int t \, dt = \frac{t^2}{2} + c = \frac{(\log x)^2}{2} + c \quad \textbf{Ans.}$$

4. Let, $I = \int \frac{x+2}{x+4} dx$

$$= \int \frac{(x+4)-2}{x+4} dx$$

$$= \int \left[1 - \frac{2}{(x+4)}\right] dx$$

$= x - 2.\log|x+4| + C$ **Ans.**

5. Let, $I = \int \frac{\sin^2 x}{1+\cos x} dx = \int \frac{1-\cos^2 x}{1+\cos x} dx$

$$= \int \frac{(1+\cos x)(1-\cos x)}{1+\cos x} dx = \int (1-\cos x) dx$$

$$= \int dx - \int \cos x \, dx = x - \sin x + C$$

where C is the constant of integration. **Ans.**

6. $$\int_0^{\pi/2} \cos^3 x \, dx = \int_0^{\pi/2} \left(\frac{3\cos x + \cos 3x}{4}\right) dx$$

$[\because \cos 3x = 4\cos^3 x - 3\cos x]$

$$= \frac{3}{4}\int_0^{\pi/2} \cos x \, dx + \frac{1}{4}\cdot\int_0^{\pi/2} \cos 3x \, dx$$

$$= \frac{3}{4}\cdot[\sin x]_0^{\pi/2} + \frac{1}{4}\left[\frac{\sin 3x}{3}\right]_0^{\pi/2} = \left(\frac{3}{4} - \frac{1}{12}\right) = \frac{8}{12} = \frac{2}{3}$$

Ans.

7. Let, $I = \int_0^{\pi/4} (\tan x - x) \tan^2 x \, dx$

$$\Rightarrow I = \int_0^{\pi/4} (\tan x - x)(\sec^2 x - 1)\, dx$$

Now, let $\tan x - x - 2$ then $(\sec^2 x - 1)\, dx = dz$

$$\therefore I = \int z dz = \frac{z^2}{2} = \frac{1}{2}(\tan x - x)^2$$

So, $I = \frac{1}{2}[\tan x - x]_0^{\pi/4}$

$$= \frac{1}{2}\left[\left(\tan\frac{\pi}{4} - \tan 0\right)^2 - \left(\frac{\pi}{4} - 0\right)^2\right]$$

$$\Rightarrow I = \frac{1}{2}\left[\left(1 - 0 - \frac{\pi}{4} + 0\right)^2\right]$$

$$= \frac{1}{2}\left(1 - \frac{\pi}{4}\right)^2 = \frac{1}{32}(\pi - 4)^2 \quad \textbf{Ans.}$$

8. Let, $I = \int_0^{2\pi} |\sin x| dx = \int_0^{\pi} |\sin x| dx + \int_{\pi}^{2\pi} |\sin x| dx$

$$= \int_0^{\pi} \sin x dx - \int_{\pi}^{2\pi} \sin x dx = [-\cos x]_0^{\pi} - [-\cos x]_{\pi}^{2\pi}$$

$= -(\cos \pi - \cos 0) + (\cos 2\pi - \cos \pi)$

$= -(-1 - 1) + [1 - (-1)] = 4$ **Ans.**

9. Let $f(x) = y \Rightarrow y = 3x - 4$

$$\Rightarrow x = \frac{y+4}{3} \Rightarrow f^{-1}(y) = \frac{y+4}{3}$$

$$\therefore f^{-1}(x) = \frac{x+4}{3}$$

10. $$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}, \; A^2 = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times3+(-5)\times(-4) & 3\times(-5)+(-5)\times2 \\ -4\times3+2\times(-4) & -4\times(-5)+2\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

$$5A = \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix}$$

$\therefore$ $A^2 - 5A - 14I$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} = 0$$

11. $$A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 4-3 \\ 3+2 & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

12. $$\Delta = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

Operate : $C_1 \to C_1 - C_2$; $C_2 \to C_2 - C_3$

$$\Delta = xyz \begin{vmatrix} 0 & 0 & 0 \\ x-y & y-z & z \\ x^3-y^2 & y^2-z^2 & z^2 \end{vmatrix}$$

$$= xyz \begin{vmatrix} x-y & y-z \\ x^2-y^2 & y^2-z^2 \end{vmatrix}$$

$$= xyz\,(x-y)\,(y-z) \begin{vmatrix} 1 & 1 \\ x+y & y+z \end{vmatrix}$$

$$= xyz\,(x-y)\,(y-z)\,(z-x).$$

13. Let $y = \cos\sqrt{x}$

Now $$\frac{dy}{dx} = \frac{d(\cos\sqrt{x})}{dx}$$

$$= \frac{d\cos\sqrt{x}}{d\sqrt{x}} \cdot \frac{d\sqrt{x}}{dx}$$

$$= -\sin\sqrt{x} \cdot \frac{1}{2\sqrt{x}} = -\frac{1}{2\sqrt{x}} \sin\sqrt{x}$$ **Ans.**

14. $$\frac{dy}{dx} = \frac{d}{dx}(\sin\sqrt{\cos x})$$

$$= \frac{d\sin\sqrt{\cos x}}{d\sqrt{\cos x}} \cdot \frac{d\sqrt{\cos x}}{d\cos x} \cdot \frac{d(\cos x)}{dx}$$

$$= \cos\sqrt{\cos x} \cdot \frac{1}{2\sqrt{\cos x}} \cdot (-\sin x)$$

$$= -\frac{\sin x \cos\sqrt{\cos x}}{2\sqrt{\cos x}}$$ **Ans.**

15. Given $x + y = \sin(xy)$

Differentiating w.r. to x, we get

$$1 + \frac{dy}{dx} = \frac{d}{dx}(\sin(xy)) = \frac{d}{dx}\sin(xy) \cdot \frac{d}{dx}(xy)$$

$$= \cos(xy) \cdot \left(1 \cdot y + x\frac{dy}{dx}\right)$$

$$= y\cos(xy) + x\cos(xy)\frac{dy}{dx}$$

$$\therefore \quad [1 - x\cos(xy)]\frac{dy}{dx} = y\cos(xy) - 1$$

$$\therefore \quad \frac{dy}{dx} = \frac{y\cos(xy) - 1}{1 - x\cos(xy)}$$ **Ans.**

16. $$\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$$

$$\frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right)$$

Putting $y = Vx$

$$\frac{dy}{dx} = V + x\frac{dy}{dx}$$

$$v + x\frac{dv}{dx} = v - \operatorname{cosec} v$$

$$\Rightarrow x\frac{dv}{dx} = -\operatorname{cosec} v$$

$$\Rightarrow \frac{dv}{\operatorname{cosec} v} = \frac{dx}{x}$$

$$\Rightarrow -\int \sin v\, dv = \int \frac{1}{x} dx$$

$$\Rightarrow -(-\cos v) = \log|x| + c$$

$$\Rightarrow \cos\frac{y}{x} = \log|x| + c$$

Putting $y = 0$ and $x = 1$

$$\Rightarrow \cos\frac{0}{1} = \log|1| + c$$

$$\cos\frac{y}{x} = \log|x| + 1$$

$$\Rightarrow 1 = 0 + c \Rightarrow c = 1$$

17. $(x^3 + y^3)\, dy - x^2y\, dx = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2y}{x^3+y^3} \quad \text{... (i)}$$

the given differential equation is homogoneous

Let $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

from eq. (i)

$$v + x\frac{dv}{dx} = \frac{vx^3}{x^3 + v^3x^3}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x^3 \times v}{x^3(1+v^3)}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-v-v^4}{1+v^3}$$

$$\Rightarrow \frac{(1+v^3)\,dv}{v^4} = -\frac{1}{x}dx$$

$$\Rightarrow \int\left(\frac{1}{x^4}+\frac{1}{v}\right)dv = -\int\frac{1}{x}dx$$

$$\Rightarrow \frac{-1}{3v^3} + \log|v| = -\log|x| + c$$

$$\Rightarrow \frac{-1}{3v^3} + \log|v| + \log|x| = c$$

$$\Rightarrow \frac{1}{3v^3} + \log|vx| = c$$

$$\Rightarrow \frac{x^3}{3\times y^3} + \log(y) = c$$

18. For the given three vectors to be coplanar, their scalar triple product is zero.

$$\Rightarrow \begin{vmatrix} 1 & 2 & -3 \\ p & -1 & 1 \\ 3 & -4 & 5 \end{vmatrix} = 0$$

$\Rightarrow$ $1(-5+4) - 2(5p-3) - 3(-4p+3) = 0$

$\Rightarrow$ $-1 - 10p + 6 + 12p - 9 = 0$

$\Rightarrow$ $2p - 4 = 0 \quad \therefore \quad p = 2$

19. Here $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is 60°.

Now $\vec{a}.\vec{b} = |\vec{a}|.|\vec{b}|.\cos 60°$

$$= \sqrt{3}\times 2\times\frac{1}{2} = \sqrt{3}$$

20. The given lines are

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} \qquad ...(1)$$

and $$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} \qquad ...(2)$$

Any point on line (1) is (3λ + 1, 2λ –1, 5λ + 1)

For point on line (2) is (4λ′ + 2, 3λ′□+ 1, – 2λ′ –1)

For lines (1) and (2) to intesect, these points must concide for some values of λ and λ′.

$\therefore$ 3λ + 1 = 4λ′ + 2 ⇒ 3λ – 4λ′ = 1 ...(3)

2λ – 1 = 3λ′ + 1 ⇒ 2λ – 3λ′ = 2 ...(4)

5λ + 1 = –2λ′ – 1 ⇒ 5λ + 2λ′ = –2 ...(5)

Solving equations (3) and (4), we get

λ = – 5, λ′ = –4

These values of λ and λ′ do not satisfy equation (5)

$\therefore$ Lines (1) and (2) do not intersect.

21. Let A = the event of getting an odd number of first die

and, B = the event of getting a multiple of 3 on the second die.

Then A = {1, 3, 5} and B {3, 6}

$$\therefore \quad P(A) = \frac{3}{6} = \frac{1}{2} \text{ and } P(B) = \frac{2}{6} = \frac{1}{3}$$

Now, **required probability**

$= P(A \cap B)$

$= P(A) \cdot P(B)$

[$\because$ A and B are independent]

$$= \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6} \text{ **Ans.**}$$

22. Let, $\tan^{-1} x = \theta$, then $\tan\theta = x$

But $\tan\theta = \cot\left(\frac{\pi}{2}-\theta\right)$

$$\therefore \quad \cot\left(\frac{\pi}{2}-\theta\right) = x$$

$$\Rightarrow \cot^{-1}x = \frac{\pi}{2} - \theta$$

$$\Rightarrow \theta + \cot^{-1}x = \frac{\pi}{2}$$

$$\therefore \quad \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$$ **Proved**

23. $\because$ cos (A + B)= cos A cos B – sin A sinB

$= \cos A\cos B - \sqrt{(1-\cos^2 A)(1-\cos^2 B)}$

Let, $\cos A = x \Rightarrow A = \cos^{-1}x$ and $\cos B = y \Rightarrow B = \cos^{-1}y$

$$\therefore \quad \cos(A+B) = \left|xy - \sqrt{(1-x^2)(1-y^2)}\right|$$

$$\Rightarrow \quad A + B = \cos^{-1}\left|xy - \sqrt{(1-x^2)(1-y^2)}\right|$$

$$\therefore \quad \cos^{-1}x + \cos^{-1}y = \cos^{-1}\left|xy - \sqrt{(1-x^2)(1-y^2)}\right|.$$

Proved

24. L.H.S. = $4(\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5})$

$$= 4\left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}\right)$$

$$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= 4\tan^{-1}\left(\frac{5}{6}\times\frac{6}{5}\right) = 4\tan^{-1}(1)$$

$$= 2\cdot 2\tan^{-1}(1) = 2\tan^{-1}\frac{2.1}{1-(1)^2}$$

$$\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$= 2\tan^{-1}(0)$

$= \pi$ **proved**

25. $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$

$$= \frac{\pi}{4} + \pi - \frac{\pi}{3} = \frac{\pi}{4} + \frac{2\pi}{3} = \frac{11}{12}\pi$$

26. Let $y = \sin[\cos(x^2)]$

Then $\frac{dy}{dx} = \frac{d\sin(\cos x^2)}{dx}$

$$= \frac{d\sin(\cos x^2)}{d\cos x^2}\cdot\frac{d\cos x^2}{dx^2}\cdot\frac{dx^2}{dx}$$

$= \cos(\cos x^2)\cdot(-\sin x^2)\cdot 2x$

$= -2x\sin x^2\cos(\cos x^2)$ **Ans.**

27. Let $\vec{a} = (5,-4,2) = 5\vec{i} - 4\vec{j} + 2\vec{k}$

and $\vec{b} = (2, 1, -3) = 2\vec{i} + \vec{j} - 3\vec{k}$

Now, $\vec{a}\cdot\vec{b} = (5\vec{i} - 4\vec{j} + 2\vec{k})\cdot(2\vec{i} + \vec{j} - 3\vec{k})$

$= 5\times 2 + (-4)\times 1 + 2\times(-3)$

$= 10 - 4 - 6$

$= 10 - 10$

$= 0$

Hence, the given vectors are perpendicular. **Proved.**

28. $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$, and $P(A\cap B) = \frac{1}{4}$

$P(A') = 1 - P(A) = 1 - \frac{3}{8} = \frac{5}{8}$

$P(B') = 1 - P(B) = 1 - \frac{1}{2} = \frac{1}{2}$

and $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$$= \frac{3}{8} + \frac{1}{2} - \frac{1}{4} = \frac{5}{8}$$

$\therefore$ $P(A'\cap B') = P(A\cup B')$ [From D'Morgan Law]

$$= 1 - P(A\cup B) = 1 - \frac{5}{8} = \frac{3}{8}$$

$$\therefore\quad P\left(\frac{A'}{B'}\right) = \frac{P(A'\cap B')}{P(B')} = \frac{3}{8}\times\frac{2}{1} = \frac{3}{4}$$

$$P\left(\frac{B'}{A'}\right) = \frac{P(A'\cap B')}{P(A')} = \frac{3}{8}\times\frac{8}{5} = \frac{3}{5}$$ **Ans.**

29. $y = \frac{\sin^2 x}{\sqrt{\cos x}} = \frac{1-\cos^2 x}{\sqrt{\cos x}}$

$$= \frac{1}{\sqrt{\cos x}} - \cos^{3/2}x$$

$$= \frac{-1}{2\sqrt{\cos x}}\times\sin x + \frac{3}{2}\times\sqrt{\cos x}\times\sin x$$

$$= \frac{\sin x}{2}\left(3\sqrt{\cos x} - \frac{1}{\sqrt{\cos x}}\right)$$

$$= \frac{\sin x(3\cos x - 1)}{2\sqrt{\cos x}}$$ **Ans.**

30. $y = \sin(\log x)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\{\sin(\log x)\} = \cos(\log x)\cdot\frac{1}{x} = \frac{\cos(\log x)}{x}$$ **Ans.**

31. We have, $(\cos x)^y = (\sin y)^x$

Taking logarithms on both sides, we get

$y\log(\cos x) = x\log(\sin y)$ ... (1)

Differentiating eqn. (1) w.r.t. x we get

$$\Rightarrow y.\frac{d}{dx}\log(\cos x) + \log\cos x.\frac{dy}{dx}$$

$$= x\frac{d}{dx}\log(\sin y)\frac{dy}{dx} + \log\sin y.1$$

$$\Rightarrow y.\frac{1}{\cos x}(-\sin x) + \log\cos x.\frac{dy}{dx}$$

$$= \frac{x.\cos y}{\sin y}.\frac{dy}{dx} + \log\sin y$$

$$\Rightarrow (\log\cos x - x\cot y)\frac{dy}{dx} = \log\sin y + y\tan x$$

$$\frac{dy}{dx} = \frac{\log\sin y + y\tan x}{\log\cos x - x\cot y}$$

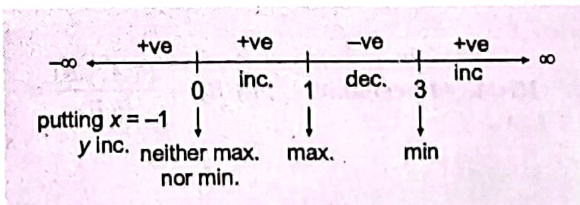
32. Let $y = x^5 - 5x^4 + 5x^3 - 1$ ...(1)

$\therefore$ $\frac{dy}{dx} = 5x^4 - 20x^3 + 15x^2 = 5x^2(x^2 - 4x + 3)$

$= 5x^2(x-1)(x-3)$ ...(2)

Now, $5x^2(x-1)(x-3) = 0 \Rightarrow x = 0, 1, 3$

Sing scheme for $\frac{dy}{dx}$ *i.e.*, for $5x^2(x-1)(x-3)$ is



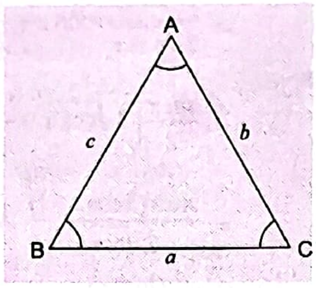
y hax maximum value at $x = 1$ and minimum value at $x = 3$.
At $x = 0$, y has neither maximum nor minimum value.
Maximum value of $y = 1 - 5 + 5 - 1 = 0$
Manimum value of $y = 243 - 5 \times 81 + 5 \times 27 - 1 = -28$

33. Area of triangle ABC

$$= \frac{1}{2}|\vec{BC} \times \vec{BA}|$$

$$= \frac{1}{2}|ac \sin B\ \hat{n}|$$

$$= \frac{1}{2} ac \sin B$$



$$= \frac{1}{2} a \frac{c}{\sin C} \sin B \sin C$$

$$= \frac{1}{2} a \frac{a}{\sin A} \sin B \sin C \quad \text{[By sine formula]}$$

$$= \frac{a^2 \sin B \sin C}{2 \sin A} \quad \textbf{Proved}$$

34. The direction ratio of the line is $-3, -2, 6$
Since dc's are proportional to dr's

$$\therefore \frac{l}{3} = \frac{m}{-2} = \frac{n}{6} = \frac{1}{\sqrt{9+4+36}}$$

$$\frac{1}{3} = \frac{m}{-2} = \frac{n}{6} = \frac{1}{7}$$

$$\therefore l = \frac{3}{7}, m = \frac{-2}{7}, n = \frac{6}{7}$$

Hence dc's are $\frac{3}{7}, \frac{-2}{7}$ and $\frac{6}{7}$ **Ans.**

35. When two coins are tossed, let S be the sample space and

A = The event of coming up of two heads

and B = The event of coming up of at least one head

Then $S = \{(H, H), (H, T), (T, H), (T, T)\}$
$A = \{(H, H)\}$ and $B = \{(H, H), (H, T), (T, H)\}$
$\therefore$ $n(S) = 4$, $n(B) = 3$, $n(A) = 1$
Also $A \cap B = \{(H, H)\}$ $\therefore$ $n(A \cap B) = 1$

Now $P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$ and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{4}$

$\therefore$ **Required probability,** $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

Ans.

36. Let the given determinant be Δ. Then,

$$\Delta = \begin{bmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{bmatrix}$$

$$= \begin{vmatrix} (b+c)^2 - a^2 & 0 & a^2 \\ 0 & (c+a)^2 - b^2 & b^2 \\ c^2 - (a+b)^2 & c^2 - (a+b)^2 & (a+b)^2 \end{vmatrix}$$

$[C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3]$

$$= \begin{vmatrix} (a+b+c)(b+c-a) & 0 & a^2 \\ 0 & (a+b+c)-(c+a-b) & b^2 \\ (a+b+c)(c-a-b) & (a+b+c)(c-a-b) & (a+b)^2 \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} (b+c-a) & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$$

[taking $(a + b + c)$ common from C_1 and C_2 both]

$$= (a+b+c)^2 \begin{vmatrix} (b+c-a) & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$$

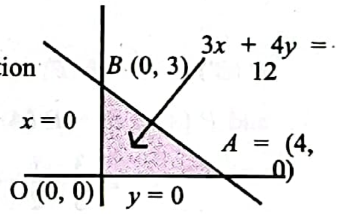
$[R_3 \to R_3 - (R_1 + R_2)]$

$= (a + b + c)^2 [(b + c - a) \{(c + a - b).2ab + 2ab^2\} + a^2 \{0 + 2b(c + a - b)\}]$
$= (a + b + c)^2 [(b + c - a).2ab\{(c + a - b)\} + 2a^2b (c + a - b)]$
$= 2ab(a + b + c) \{(b + c - a) (c + a) + a(c + a - b)\}$
$= 2ab(a + b + c)^2. \{bc + ab + c^2 + ac - ac - a^2 - ab\}$
$= 2ab (a + b + c)^2 \{bc + c^2 + ac\}$
$= 2abc (a + b + c)^3$

Hence, $\Delta = 2abc (a + b + c)^3$; **Ans.**

37. Given objective function, $z = 2x + 3y$
And constraints are $3x + 4y \le 12$
and $x \ge 0, y \ge 0$
$\because$ $3x + 4y \le 12$
Corresponding equation
$3x + 4y = 12$

$$\frac{3x}{12} + \frac{4y}{12} = \frac{12}{12}$$



$$\Rightarrow \frac{x}{4} + \frac{y}{3} = 1 \quad \text{...(i)}$$

The lines cuts intercepts 4 and 3 on the x-axis and y-axis respectively.
Now $x \ge 0$ and $y \ge 0$ taken together represent first quadrant.
Check for (0, 0)
$\because$ $3x + 4y \le 12$
$\Rightarrow$ $3.0 + 4.0 \le 12$
$\Rightarrow$ $0 \le 12$ [True]
Critical points O (0, 0), A (4, 0) and B (0, 3).

Critical point	value of $z = 2x + 3y$
O (0, 0)	$2.0 + 3.0 = 0$
A (4, 0)	$2.4 + 3.0 = 8$
B (0, 3)	$2.0 + 3.3 = 9$

Hence $z = 2x + 3y$ is maximum on B (0, 3). **Ans.**

38. Let, $I = \int_0^{\pi/2} \log \tan x \, dx$ ...(i)

Then, $I = \int_0^{\pi/2} \log\left[\tan\left(\frac{\pi}{2} - x\right)\right] dx$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

or, $I = \int_0^{\pi/2} \log(\cot x) dx$

$= \int_0^{\pi/2} \log\left(\frac{1}{\tan x}\right) dx$

$= -\int_0^{\pi/2} \log \tan x \, dx = -1$

$\therefore$ $I = -I$ or $2I = 0$ or $I = 0$

Hence, $\int_0^{\pi/2} \log \tan x \, dx = 0$; **Ans.**

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