MODEL PAPER – 3

Time : 3 Hours + 15 Minutes] **[Total Marks : 100**

Instructions to the Candidates :

1. Candidates are required to give their answers in their own words as far as practicable.
2. Figures in the right hand margin indicate full marks.
3. 15 Minutes of extra time has been allotted for the candidates to read the questions carefully.
4. This question paper is divided into two sections—**SECTION – A** and **SECTION – B**.
5. In **SECTION – A** there are **100 Objective Type Question,** out of which only 50 objective question be answered. Darken the circle with blue/black ball pen against the correct option on OMR Sheet provided to you. Do not use **Whitener/Liquid/Blade/Nail** on OMR paper; otherwise the result will be invalid.
6. In **SECTION – B**, there are **30 Short Answer Type Questions** (each carrying 2 marks), out of which any 15 questions are be answered.
   Apart from this, there are **8 Long Answer Type Question** (Each Carrying 5 marks), out of which 4 Questions are to be answered.
7. Use of any electronic device is prohibited.

SECTION – A : Objective Type Questions

Direction : *There are 100 Objective Type Questions, out of which only 50 objective questions to be answered. Mark the correct option on the OMR Answer Sheet.* 50 × 1 = 50

1. If $A = \{1, 2\}$, $B = \{a, b, c\}$ then total number of functions from A to B is :
   (A) 9 (B) 12
   (C) 64 (D) None of these

2. $|\vec{i} - 2\vec{j} + 2\vec{k}| =$
   (A) 3 (B) 6
   (C) 7 (D) 5

3. $(10\vec{i} + \vec{j} + \vec{k}) \times (-4\vec{i} + 7\vec{j} - 11\vec{k}) =$
   (A) $-18\vec{i} - 106\vec{j} + 74\vec{k}$ (B) $18\vec{i} - 106\vec{j} - 74\vec{k}$
   (C) $18\vec{i} + 106\vec{j} + 74\vec{k}$ (D) $5\vec{i} - 6\vec{j} - 7\vec{k}$

4. The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2y}{x} = 5x^2$ is :
   (A) $\frac{2}{x}$ (B) $2e^x$
   (C) $2\log x$ (D) x^2

5. $\int \frac{3x^2 + 2}{x^3 + 2x} dx =$
   (A) $\sin^{-1}(x^3 + 2x) + k$ (B) $\tan^{-1}(3x^2 + 2) + k$
   (C) $\log |3x^2 + 2| + k$ (D) $\log |x^3 + 2x| + k$

6. $x \in [-1\ 1], \sin[2(\sin^{-1}x + \cos^{-1}x)] =$
   (A) 0 (B) 1 (C) –1 (D) $\frac{1}{2}$

7. $P(E) = \frac{3}{7}, P(F) = \frac{5}{7}, P(E \cup F) = \frac{6}{7} \Rightarrow P(E \cap F) =$
   (A) $\frac{4}{7}$ (B) $\frac{2}{7}$
   (C) $\frac{1}{7}$ (D) $\frac{3}{7}$

8. The direction ratios of the straight line $\frac{x+1}{3} = \frac{y-2}{3} = \frac{z-5}{6}$ are :
   (A) 1, –2, 5 (B) 3, 2, 5
   (C) 3, 3, 6 (D) 1, 3, 5

9. $\int \frac{3\cos x - 2\sin x}{2\cos x + 3\sin x} dx =$
   (A) $2\cos x + 3\sin x + k$ (B) $\log|2\cos x + 3\sin x| + k$
   (C) $\tan^{-1}\left(3\sin\frac{x}{2}\right) + k$ (D) $2\tan\frac{x}{2} + k$

10. $\frac{d}{dx}(e^x + \cos 5x) =$
    (A) $e^x + \cos 5x$ (B) $e^x + 5\sin 5x$
    (C) $e^x - 5\sin 5x$ (D) $e^x - 5\cos 5x$

11. $\frac{d}{dx}(\sin 2x + e^x - \cos x) =$
    (A) $\cos 2x + e^x - \sin x$ (B) $2\cos 2x + e^x + \sin x$
    (C) $2\cos 2x + e^x - \sin x$ (D) $-2\cos 2x + e^x + \sin x$

12. $3\begin{bmatrix} 7 & -2 \\ 8 & 0 \end{bmatrix} =$
    (A) $\begin{bmatrix} 21 & -6 \\ 8 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & -2 \\ 24 & 0 \end{bmatrix}$
    (C) $\begin{bmatrix} 21 & -6 \\ 24 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 21 & -2 \\ 8 & 0 \end{bmatrix}$

13. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} =$

(A $\begin{bmatrix} 4 & 6 \\ 25 & 35 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 15 \\ 10 & 35 \end{bmatrix}$ (C) $[19\ 45]$ (D) $\begin{bmatrix} 19 \\ 45 \end{bmatrix}$

14. $[2a-7\ \ 1] = [a\ \ b-1] \Rightarrow (a, b) =$

(A) (1, 7) (B) (2, 7) (C) (7, 2) (D) (2, 3)

15. The operation is * is defined as $a * b = 2a + b$, then $(2 * 3) * 4$ is :

(A) 18 (B) 17 (C) 19 (D) 21

16. $\cot^{-1}(-x) =$

(A) $-\cot^{-1} x$ (B) $\cot^{-1} x$

(C) $\pi + \cot^{-1} x$ (D) $\pi - \cot^{-1} x$

17. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is :

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{-\pi}{6}$ (D) $\frac{\pi}{6}$

18. If $\vec{a} = \vec{i} + \vec{j} + 2\vec{k}$ and $b = 3\hat{i} + 2\hat{j} - \hat{k}$ then the value of $(\vec{a} + 3\vec{b})(2\vec{a} - \vec{b}) =$

(A) 15 (B) –15 (C) 18 (D) –18

19. If $|\vec{a}| = \sqrt{26}$, $|b| = 7$ and $|\vec{a} \times \vec{b}| = 35$ then $\vec{a}.\vec{b} =$

(A) 8 (B) 7 (C) 9 (D) 12

20. $\int \frac{dx}{1+x^2} =$

(A) $\cot^{-1} x + C$ (B) $\tan^{-1} x + C$

(C) $\sec^{-1} x + C$ (D) $\mathrm{cosec}^{-1} + C$

21. $\int_a^b e^x dx =$

(A) $(e^b - e^a)$ (B) e^{a-b}

(C) $e^{-(a-b)}$ (D) e^{a+b}

22. $\frac{d}{dx}(e^{\cos x}) = ?$

(A) $(\sin x)e^{\cos x}$ (B) $-(\sin x)e^{\cos x}$

(C) $(\cos x)e^{\cos x}$ (D) $-(\cos x)e^{\cos x}$

23. $\frac{d}{dx}\{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}\} =$

(A) $\frac{\pi}{2}$ (B) 0 (C) 1 (D) $\sqrt{x}\cdot\frac{\pi}{2}$

24. Which of the following is the unit matrix of order 3 × 3 ?

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

25. If A and B are two independent events then $P(A \cap B) =$

(A) P (A) × P (B) (B) P(A/B)

(C) P(A) + P(B) (D) (P) + P(B) – $P(A \cap B)$

26. If S be the sample space and E be the event that P(E) = ..

(A) $\frac{n(E)}{n(S)}$ (B) $\frac{n(S)}{n(E)}$ (C) $n(E)$ (D) $n(S)$

27. Let a, b, c be the direction ratios of a line then direction each as are :

(A) $\frac{a}{\sqrt{\Sigma a^2}}, \frac{b}{\sqrt{\Sigma a^2}}, \frac{c}{\sqrt{\Sigma a^2}}$ (B) $\frac{1}{\sqrt{\Sigma a^2}}, \frac{1}{\sqrt{\Sigma a^2}}, \frac{1}{\sqrt{\Sigma a^2}}$

(C) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ (D) $\frac{a}{\sqrt{\Sigma a^2}}, \frac{b}{\sqrt{\Sigma b^2}}, \frac{c}{\sqrt{\Sigma c^2}}$

28. A lines is passing through (α, β, γ) and its direction cosines are *l*, *m*, *n* then the equations of the line are:

(A) $\frac{x}{l} = \frac{\gamma}{n} = \frac{z}{n}$ (B) $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$

(C) $\frac{x+\alpha}{l} = \frac{y+\beta}{m} = \frac{z+\gamma}{n}$ (D) $\frac{x-\alpha}{l} = \frac{y+\beta}{m} = \frac{z-\gamma}{n}$

29. $\int x^2 \sin x^3\, dx = ?$

(A) $-\frac{1}{3}\cos x^3 + k$ (B) $\frac{1}{3}\cos x^3 + k$

(C) $\frac{1}{3}\sin x^3 + k$ (D) $-\frac{1}{3}\sin x^3 + k$

30. $\int \mathrm{cosec}\, x\, dx = ?$

(A) $\log\left|\tan\frac{x}{2}\right| + k$ (B) $\log\left|\cot\frac{x}{2}\right| + k$

(C) $\log\left|\sin\frac{x}{2}\right| + k$ (D) $\log\left|\cos\frac{x}{2}\right| + k$

31. If $(a + b + c)$ is positive and not all a, b, c are equal then the value of the following determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$

(A) Positive (B) Negative

(C) 0 (D) None of these

32. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 30 & 3 & i \end{vmatrix} = x + iy$, then

(A) $x = 3, y = 1$ (B) $x = 1, y = 3$

(C) $x = 0, y = 3$ (D) $x = 0, y = 0$

33. $\vec{i} \times \vec{k} =$

(A) 1 (B) $\vec{0}$ (C) $\vec{j}$ (D) $-\vec{j}$

34. $\vec{i}.(\vec{j} \times \vec{k}) =$

(A) 0 (B) 1 (C) –1 (D) 2

35. The integrating factor (I.F.) of the differential equation $\cos^2 x\frac{dy}{dx} + y = \tan x$ is

(A) $e^{\tan x}$ (B) $e^{\cot x}$ (C) $e^{\sin x}$ (D) $e^{\cos x}$

36. $\frac{d}{dx}\cdot(\log x^n) =$

(A) $\frac{1}{x^n}$ (B) n (C) $\frac{1}{x}$ (D) $\frac{n}{x}$

37. $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to :

(A) $-\frac{\pi}{2}$ (B) π (C) 0 (D) $2\sqrt{3}$

38. $\cot(\tan^{-1} x + \cot^{-1} x) = ?$

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{\pi}{4}$

39. $\int \operatorname{cosec}^2 x \, dx = ?$

(A) $\tan x + c$ (B) $-\cot x + c$
(C) $2\operatorname{cosec} x + c$ (D) $-2\operatorname{cosec} x + c$

40. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^9 x \, dx = ?$

(A) -1 (B) 0 (C) 1 (D) $\frac{\pi}{2}$

41. $(\vec{a} \times \vec{a}) \cdot \vec{b}$

(A) 1 (B) -1 (C) 0 (D) 2

42. $\vec{i} \cdot \vec{j} =$

(A) 1 (B) 0 (C) k (D) $-\vec{k}$

43. $\int 1.dx =$

(A) $x + k$ (B) $1 + k$
(C) $\frac{x^2}{2} + k$ (D) $\log x + k$

44. $\int \frac{dx}{\sqrt{x}} =$

(A) $\sqrt{x} + k$ (B) $2\sqrt{x} + k$
(C) $x + k$ (D) $\frac{2}{3}x^{3/2} + k$

45. $\begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & 7 \\ 0 & 0 & 5 \end{bmatrix} =$

(A) 40 (B) 0 (C) 3 (D) 25

46. Solution of $xdx + \frac{xdy - ydx}{x^2 + y^2} = 0$ is :

(A) $\frac{x^2}{2} + \tan^{-1}\frac{x}{y} = k$ (B) $\frac{x^2}{2} + \tan^{-1}\frac{y}{x} = k$

(C) $\frac{x^2}{2} - \tan^{-1}\frac{x}{y} = k$ (D) $\frac{x^2}{2} - \tan^{-1}\frac{y}{x} = k$

47. If $F : R \to R$ such that $f(x) = 5x + 4$ then which of the following is equal to $f^{-1}(x)$.

(A) $\frac{x-5}{4}$ (B) $\frac{x-y}{5}$ (C) $\frac{x-4}{5}$ (D) $\frac{x}{4} - 5$

48. If an operation is defined by $a * b = a^2 + b^2$, then $(1 * 2) * 6$ is :

(A) 12 (B) 28
(C) 61 (D) None of these

49. Matrices A and B will be inverse of each other only if :

(A) $AB = BA$ (B) $AB = BA = 0$
(C) $AB = 0, BA = I$ (D) $AB = BA = I$

50. If two events are independent, then :

(A) they must be mutually exclusive
(B) the sum of their probabilities must be equal to 1
(C) 'A' and 'B' both are correct
(D None of the above is correct

51. $1 - P(A' \cap B') =$

(A) $P(A \cap B)$ (B) $P(A \cup B)$
(C) $P(A)$ (D) $P(B)$

52. $\int x^3 dx =$

(A) $\frac{x^4}{4} + k$ (B) $3x^2 + k$

(C) $k + x^4$ (D) $k + \frac{x^3}{4}$

53. $\int \cos x \, dx =$

(A) $-\sin x + k$ (B) $\cos x + k$
(C) $\sin x + k$ (D) $-\cos x + k$

54. If $x^m y^n = (x + y)^{m+n}$, then $\frac{dy}{dx}$ is equal to :

(A) $\frac{x+y}{xy}$ (B) xy (C) $\frac{x}{y}$ (D) $\frac{y}{x}$

55. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) =$

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{4\pi}{3}$ (D) $\frac{\pi}{3}$

56. $\sin^{-1}(1 - x) - 2\sin^{-1}x = \frac{\pi}{2}$ then $x = ?$

(A) $0, \frac{1}{2}$ (B) $1, \frac{1}{2}$ (C) $\frac{1}{2}$ (D) 0

57. If A, B are symmetric matrices of same order then $AB - BA$ is :

(A) skew-symmetric matrix (B) symmetric matrix
(C) zero matrix (D) identity matrix

58. The degree of the equation $\left(\frac{dy}{dx}\right)^4 + 3y\left(\frac{d^2y}{dx^2}\right) = 0$ is :

(A) 4 (B) 1 (C) 2 (D) 3

59. If A and B are two events such that $P(A / B) = P(B / A) \neq 0$, then

(A) $A \subset B$ but $A \neq B$ (B) $A = B$
(C) $A \cap B = \phi$ (D) $P(A) = P(B)$

60. If $P(A) = \frac{1}{2}, P(B) = 0$, then $P(A/B)$ is :

(A) 0 (B) $\frac{1}{2}$

(C) 1 (D) not defined

61. Standard equation of the plane in intercept form is :

(A) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ (B) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

(C) $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} = 0$ (D) None of these

62. The direction ratios of the line joining the points (x, y, z) and (x_2, y_2, z_2) are :

(A) $x_1 + x_2, y_1 + y_2, z_1 + z_2$

(B) $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

(C) $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}$

(D) $x_2 - x_1, y_2 - y_1, z_2 - z_1$

63. The position vector of the point (1, 0, 2) is :

(A) $\vec{i}+\vec{j}+2\vec{k}$ (B) $\vec{i}+2\vec{j}$

(C) $\vec{2}+3\vec{k}$ (D) $\vec{i}+2\vec{k}$

64. The modulus of $7\vec{i}-2\vec{j}+\vec{k}$:

(A) $\sqrt{10}$ (B) $\sqrt{55}$ (C) $3\sqrt{6}$ (D) 6

65. $\int_0^1 \frac{dx}{x+1} =$

(A) log2 (B) – log2

(C) 2log2 (D) – 2log2

66. $\int_0^3 x\,dx =$

(A) $\frac{3}{2}$ (B) 9 (C) $\frac{9}{2}$ (D) $\frac{9}{4}$

67. If $\omega \neq 1, \omega^3 = 1$ and $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$ then $x =$

(A) 1 (B) ω (C) ω^2 (D) 0

68. $\begin{vmatrix} 3 & 4 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 7 \end{vmatrix} = A$ then $|A| =$

(A) 40 (B) 50 (C) 42 (D) 15

69. $\frac{d}{dx}(\tan x^2) =$

(A) $\sec x^2$ (B) $2x\sec^2 x^2$

(C) $2x^2 \sec^2 x^2$ (D) $\frac{\sec x^2}{2x}$

70. $\frac{d}{dx}\left(\sqrt{\cot x}\right) =$

(A) $\frac{1}{2\sqrt{\cot x}}$ (B) $\sqrt{\text{cosec}^2 x}$

(C) $\frac{-\text{cosec}^2 x}{2\sqrt{\cot x}}$ (D) $\frac{\text{cosec}^2 x}{2\sqrt{\cot x}}$

71. $3\begin{bmatrix} a & b \\ c & d \end{bmatrix} =$

(A) $\begin{bmatrix} 3a & 3b \\ c & d \end{bmatrix}$ (B) $\begin{bmatrix} 3a & 3b \\ 3c & d \end{bmatrix}$ (C) $\begin{bmatrix} a & 3b \\ 3c & 3d \end{bmatrix}$ (D) $\begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix}$

72. If $A = [3 \quad 4 \quad 5]$ and $B = [1 \quad 2 \quad 1]$, then $A + B =$

(A) $[4 \quad 6 \quad 6]$ (B) $\begin{bmatrix} 4 \\ 6 \\ 6 \end{bmatrix}$

(C) $[4 \quad 6 \quad 4]$ (D) $[6 \quad 4 \quad 6]$

73. $\text{cosec}^{-1}(-x) =$

(A) $\frac{\pi}{2} - \text{cosec}^{-1} x$ (B) $\pi - \text{cosec}^{-1} x$

(C) $\text{cosec}^{-1} x$ (D) $-\text{cosec}^{-1} x$

74. $\cot^{-1}\frac{1}{x} =$ ...., $(x > 0)$

(A) $-\cot^{-1} x$ (B) $\tan^{-1}\frac{1}{x}$

(C) $\tan^{-1} x$ (D) $\cot^{-1} x$

75. $\frac{d}{dx}(\sin^{-1} x) =$

(A) $\frac{1}{\sqrt{1-x^2}}$ (B) $-\frac{1}{\sqrt{1-x^2}}$

(C) $2\sqrt{1-x^2}$ (D) $(1-x)$

76. The equation of *yz*-Plane is :

(A) $y = 0, z = 0$ (B) $x = 0$

(C) $y = 0$ (D) $x = 1$

77. $\int \frac{xe^x}{(x+1)^2} dx =$

(A) $\frac{e^x}{(x+1)^2}+C$ (B) $\frac{-e^x}{x+1}+C$

(C) $\frac{e^x}{x+1}+C$ (D) $\frac{-e^x}{(x+1)^2}+C$

78. $\int \frac{dx}{a^2+x^2} =$

(A) $\frac{1}{a}\tan^{-1}\frac{a}{x}+C$ (B) $\tan^{-1}\frac{x}{a}+C$

(C) $\frac{1}{a}\tan^{-1}\frac{x}{a}+C$ (D) $\frac{1}{a}\tan^{-1} x+C$

79. If $\vec{a}$ and $\vec{b}$ are perpendicular to each other then :

(A) $\vec{a}.\vec{b}=0$ (B) $\vec{a}\times\vec{b}=\vec{0}$

(C) $\vec{a}+\vec{b}=\vec{0}$ (D) $\vec{a}-\vec{b}=0$

80. $\vec{a}\times\vec{a} =$

(A) 1 (B) $\vec{0}$ (C) a^2 (D) a

81. Solution of the differential equation $\frac{ydx-xdy}{y^2}=0$ is :

(A) $\frac{y}{x}=k$ (B) $\frac{x}{y^2}=k$

(C) $\frac{x}{y}=k$ (D) None of these

82. The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+2\left(\frac{dy}{dx}\right)^3+9y=\sin x$ is :

(A) 3 (B) 4

(C) 2 (D) None of these

83. $\frac{d}{dx}(\sec x) =$

(A) $\sec^2 x$ (B) $\tan^2 x$

(C) $\sec x \cdot \tan x$ (D) O

84. If $y = \sin(\log x)$, then $\frac{dy}{dx} =$ ......

(A) $\frac{1}{x}\cos(\log x)$ (B) $\frac{1}{x}\sin(\log x)$

(C) 0 (D) 1

85. $\begin{vmatrix} 4 & 5 \\ 16 & 20 \end{vmatrix} =$

(A) 160 (B) 80 (C) –160 (D) 0

86. $2\begin{vmatrix} a & b \\ c & d \end{vmatrix} =$

(A) $\begin{vmatrix} 2a & b \\ c & d \end{vmatrix}$ (B) $\begin{vmatrix} a & 2b \\ c & d \end{vmatrix}$ (C) $\begin{vmatrix} a & b \\ 2c & d \end{vmatrix}$ (D) $\begin{vmatrix} 2a & b \\ 2c & d \end{vmatrix}$

87. If A and B are any two events such that P(A) + P(B) – P (A and B) = P (A) then

(A) $P\left(\frac{B}{A}\right)=1$ (B) $P\left(\frac{B}{A}\right)=0$

(C) $P\left(\frac{A}{B}\right)=1$ (D) $P\left(\frac{A}{B}\right)=0$

88. If $\vec{a} = \vec{i} + \vec{j} + 2\vec{k}$, then the corresponding unit vector $\hat{a}$ in the directon of $\vec{a}$ =

(A) $\frac{\vec{i}+\vec{j}+\vec{k}}{\sqrt{6}}$ (B) $\frac{\vec{i}+\vec{j}+2\vec{k}}{\sqrt{6}}$

(C) $\frac{\vec{i}+\vec{j}+2\vec{k}}{6}$ (D) None of these

89. The direction cosines of the vector $3\vec{i}-4\vec{j}+12\vec{k}$ is :

(A) $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$ (B) $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$

(C) $\frac{3}{\sqrt{13}}, \frac{4}{\sqrt{13}}, \frac{12}{\sqrt{13}}$ (D) $\frac{3}{\sqrt{13}}, \frac{-4}{\sqrt{13}}, \frac{12}{\sqrt{13}}$

90. $\int_0^{\pi/2} \cos x \, dx =$

(A) 1 (B) –1 (C) 0 (D) 2

91. $\int_1^3 dx =$

(A) 4 (B) 2 (C) 3 (D) $\frac{1}{2}$

92. If $f : R \to R$ is a function $f^{-1} : R \to R$ will exist if f is

(A) one-one into (B) onto

(C) one-one onto (D) many-one onto

93. If $n(A) = 3$ and $n(B) = 2$ then $n(A \times B) =$ ....

(A) 6 (B) 4 (C) 2 (D) 0

94. If $\Delta = \begin{vmatrix} 10 & 2 \\ 30 & 6 \end{vmatrix}$ than $\Delta =$

(A) 0 (B) 10 (C) 12 (D) 60

95. $\frac{d}{dx}(\tan kx) =$

(A) $\sec^2 kx$ (B) $k\sec^2 x$

(C) $\frac{\sec^2 kx}{k}$ (D) $k\sec^2 kx$

96. $\frac{d}{dx}(\log\sqrt{x}) =$

(A) $\frac{1}{2\sqrt{x}}$ (B) $\frac{1}{\sqrt{x}}$

(C) $\frac{1}{2x}$ (D) $\frac{\sqrt{x}}{2}$

97. $\int \frac{e^{2x}-1}{e^{2x}+1} dx =$

(A) $\log(e^{2x} + 1) + c$ (B) $\log(e^x + e^{-x}) + c$

(C) $\log(e^x - e^{-x}) + c$ (D) $e^{2x} + c$

98. $\int \cos x^\circ dx =$

(A) $\sin x^\circ + c$ (B) $\frac{\pi}{180}\sin x^\circ + c$

(C) $\frac{180}{\pi}\sin x^\circ + c$ (D) $\frac{\pi x}{180}\sin x^\circ + c$

99. $\vec{j} \times \vec{i} =$

(A) $\vec{k}$ (B) $-\vec{k}$ (C) 1 (D) 0

100. $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) =$

(A) 1 (B) 0

(C) $a^2 + b^2$ (D) $a^2 - b^2$

SECTION – B : Non-Objective Type Questions

SHORT ANSWER TYPE QUESTIONS

Direction : *Question Nos. 1 to 30 are of short answer type. Answer only 15 questions from these.* $15 \times 2 = 30$

1. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$**, then show that**

$$A^2 - 4A - 5I = 0$$

2. Find the vlaue of y**, if**

$$\begin{pmatrix} x-y & 2 \\ x & 5 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 3 & 5 \end{pmatrix}$$

3. Evaluate : $\begin{vmatrix} \sin 30^\circ & \cos 30^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{vmatrix}$

4. Using properties of determinants, prove the following :

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

5. Differentiate : $\tan(2x + 3)$

6. If $y = \sin\sqrt{x^2 + ax + 1}$, find $\frac{dy}{dx}$

7. If $x^3 + y^3 = \sin(x + y)$, find $\frac{dy}{dx}$

8. Solve the equation :
$(\tan^{-1}y - x)\, dx = (1 + y^2)\, dx$

9. Solve the equation :

$$(y + x)\frac{dy}{dx} = y - x$$

10. Integrate : $\int\sqrt{1-\sin 2x}\,dx$

11. Integrate : $\int \sin 4x \cos 3x\, dx$

12. Integrate : $\int \frac{\operatorname{cosec} x}{\operatorname{cosec} x + \cot x}\,dx$

13. Integrate : $\int \tan^3 x\, dx$

14. Integrate : $\int \sin^2 x \cdot \cos^2 x dx$

15. Evaluate $\int_0^{\pi/2} \sin^4 x\, dx$

16. Find the value of $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)^2}\,dx$

17. Find the value of $\int_0^{e} \frac{\cos(\log x)}{x}\,dx$

18. Find $|\vec{a}\times\vec{b}|$ if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

19. Find the angle between the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$

20. Find the angle between the lines :

$$\frac{x+1}{3} = \frac{y-1}{2} = \frac{z-2}{4}$$

and the plane $2x + y - 3z + 4 = 0$

21. If A and B are two independent events such that
$P(A \cup B) = 0.5$, $P(A) = 0.2$, find $P(B)$

22. Show : $\operatorname{cosec}^{-1}x + \sec^{-1} x = \frac{\pi}{2}$

23. Prove that :

$$\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left|xy + \sqrt{(1-x^2)(1-y^2)}\right|$$

24. Prove that :

$$\tan\left(\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2} + \frac{1}{2}\cos^{-1}\frac{1-x^2}{1+x^2}\right) = \frac{2x}{1-x^2}$$

25. Write the value of

$$\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$$

26. Differentiate : $\sin(2\sin^{-1} x)$

27. If $A = \begin{bmatrix} 1 & 1 \\ x & y \\ x^2 & y^2 \end{bmatrix}$, the find AA'

28. Find $\frac{dy}{dx}$, when $x = a\cos^2\theta$, $y = a\sin^2\theta$.

29. Find the slope at the point $(1, \sqrt{2})$ of the curve $x^2 + y^2 = 3$.

30. Prove that $(\vec{a} + \vec{b})\cdot(\vec{a} - \vec{b}) = a^2 - b^2$

LONG ANSWER TYPE QUESTIONS

Direction : *Question Nos. 31 to 38 are long answer type question. Answer any 4 questions from these.* $5 \times 4 = 20$

31. If $\sin y = x\sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

32. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the X-axis in the first quadrant.

33. Using vector method, prove that in a ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where a, b, c are the lenghts of the side opposite respectively to the angles A, B and C of ΔABC.

34. Find the value of λ, such that the line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to the plane $3x - y - 2z = 7$.

35. If A and B are two events such $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{1}{2}$, then prove that A and B are independent events.

36. Show that :

$$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$$

37. Solve the following LPP graphically
Maximize $z = 5x + 3y$
subject to $3x + 5y \le 15$
$5x + 2y \le 10$
$x, y \ge 0$

38. Evaluate : $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x})\, dx$

ANSWER WITH EXPLANATIONS

SECTION – A

OMR ANSWER-SHEET

1.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	51.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
2.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	52.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
3.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	53.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
4.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	54.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
5.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	55.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
6.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	56.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
7.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	57.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
8.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	58.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
9.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	59.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
10.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	60.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
11.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	61.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
12.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	62.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
13.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	63.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
14.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	64.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
15.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	65.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
16.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	66.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
17.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	67.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
18.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	68.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
19.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	69.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
20.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	70.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
21.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	71.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
22.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	72.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
23.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	73.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
24.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	74.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
25.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	75.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
26.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	76.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
27.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	77.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
28.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	78.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
29.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	79.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
30.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	80.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
31.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	81.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
32.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	82.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
33.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	83.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
34.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	84.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
35.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	85.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
36.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	86.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
37.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	87.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
38.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	88.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
39.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	89.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
40.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	90.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
41.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	91.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
42.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	92.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
43.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	93.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
44.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	94.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
45.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	95.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
46.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	96.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
47.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	97.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
48.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	98.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
49.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	99.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
50.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	100.	Ⓐ	Ⓑ	Ⓒ	Ⓓ

ANSWER

1. (A)	2. (A)	3. (A)	4. (D)	5. (D)
6. (A)	7. (B)	8. (C)	9. (B)	10. (C)
11. (B)	12. (C)	13. (D)	14. (C)	15. (A)
16. (D)	17. (B)	18. (B)	19. (B)	20. (B)
21. (A)	22. (B)	23. (B)	24. (B)	25. (A)
26. (A)	27. (D)	28. (B)	29. (A)	30. (A)
31. (B)	32. (C)	33. (D)	34. (B)	35. (A)
36. (D)	37. (A)	38. (A)	39. (B)	40. (B)
41. (C)	42. (B)	43. (A)	44. (B)	45. (A)
46. (B)	47. (C)	48. (C)	49. (D)	50. (D)
51. (B)	52. (A)	53. (C)	54. (D)	55. (D)
56. (D)	57. (A)	58. (B)	59. (D)	60. (D)
61. (B)	62. (D)	63. (D)	64. (C)	65. (A)
66. (C)	67. (D)	68. (C)	69. (B)	70. (C)
71. (D)	72. (A)	73. (D)	74. (C)	75. (A)
76. (B)	77. (C)	78. (C)	79. (A)	80. (B)
81. (C)	82. (C)	83. (C)	84. (A)	85. (D)
86. (D)	87. (C)	88. (B)	89. (B)	90. (A)
91. (B)	92. (C)	93. (A)	94. (A)	95. (D)
96. (C)	97. (B)	98. (C)	99. (B)	100. (D)

SECTION – B

1. $A^2 - 4A - 5I$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$- \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 0$$

2. We have $\begin{bmatrix} x-y & 2 \\ x & 5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$

By definitions of equality of matrices

$$x - y = 2 \quad \text{... (1)}$$

put $x = 3$

putting the value of x, in (1) we get

$$3 - y = 2 \Rightarrow -y = -1 \quad \therefore \quad y = 1$$

Hence $x = 3$, $y = 1$

3. Let $\Delta = \begin{vmatrix} \sin 30° & \cos 30° \\ -\sin 60° & \cos 60° \end{vmatrix}$

$= \sin 30° \cos 60° + \cos 30° \sin 60°$

$= \sin (30° + 60°)$

$= \sin 90°$

$= 1$

4. LHS = $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

(Operate : $C_1 \to C_1 - C_2$; $C_2 \to C_2 - C_3$)

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$$

$= (a-b)(b-c)[(c^2-a^2)+(bc-ab)]$

$= (a-b)(b-c)[(c-a)(c+a)+b(c-a)]$

$= (a-b)(b-c)(c-a)(a+b+c) =$ RHS.

5. $y = \tan(2x+3)$

Now $\dfrac{dy}{dx} = \dfrac{d\tan(2x+3)}{dx}$

$= \dfrac{d\tan(2x+3)}{d(2x+3)} \cdot \dfrac{d(2x+3)}{dx}$

$= \sec^2(2x+3)\cdot 2 = 2\sec^2(2x+3)$ **Ans.**

6. $\dfrac{dy}{dx} = \dfrac{d}{dx}\sin\sqrt{x^2+ax+1}$

$= \dfrac{d\sin\sqrt{x^2+ax+1}}{d\sqrt{x^2+ax+1}} \cdot \dfrac{d\sqrt{x^2+ax+1}}{d(x^2+ax+1)} \cdot \dfrac{d}{dx}$

(x^2+ax+1)

$= \cos\sqrt{x^2+ax+1}\cdot\dfrac{1}{2\sqrt{x^2+ax+1}}\cdot(2x+a)$

$= \dfrac{(2x+a)\cos\sqrt{x^2+ax+1}}{2\sqrt{x^2+ax+1}}$ **Ans.**

7. Given, $x^3 + y^3 = \sin(x+y)$

Differentiating w.r. to x, we get

$\dfrac{d}{dx}(x^3)+\dfrac{d}{dx}(y^3) = \dfrac{d}{dx}\{\sin(x+y)\}$

or $\dfrac{d}{dx}(x^3)+\dfrac{d}{dy}(y^3)\cdot\dfrac{dy}{dx} = \dfrac{d\sin(x+y)}{d(x+y)}\cdot\dfrac{d}{dx}(x+y)$

or $3x^2+3y^2\dfrac{dy}{dx} = \cos(x+y)\cdot\left(1+\dfrac{dy}{dx}\right)$

or $[3y^2-\cos(x+y)]\dfrac{dy}{dx} = \cos(x+y)-3x^2$

$\therefore \dfrac{dy}{dx} = \dfrac{\cos(x+y)-3x^2}{3y^2-\cos(x+y)}$ **Ans.**

8. $\dfrac{dx}{dy} = \dfrac{\tan^{-1}y}{1+y^2} - \dfrac{x}{1+y^2}$

$= \dfrac{dx}{xy} + \dfrac{x}{1+y^2} = \dfrac{\tan^{-1}y}{1+y^2}$

I.F. $= e\int \rho dy = e\int\dfrac{dy}{1+yz} = e^{\tan^{-1}y}$

$= \dfrac{dx}{dy}\times e^{\tan^{-1}y} + \dfrac{x}{1+y^2}\times e^{\tan^{-1}y} = \dfrac{\tan^{-1}y}{1+y^2}\times e^{\tan^{-1}y}$

$= \dfrac{d(x\times e^{\tan^{-1}y})}{dy} = \dfrac{\tan^{-1}y\times e^{\tan^{-1}y}}{1+y^2}$

$= \int d(x\times e^{\tan^{-1}y}) = \dfrac{\tan^{-1}y\times e^{\tan^{-1}y}}{1+y^2}dy$

$= x\times e^{\tan^{-1}y} = \int\dfrac{\tan^{-1}y\times e^{\tan^{-1}y}}{1+y^2}dy$

Let $\dfrac{\tan^{-1}y}{1+y^2} = t$ and $dy = dt$ then, $x\times e^{\tan^{-1}y} = \int t\times e^t + c$

$= x\tan^{-1}y = t\times e^t - \int 1\times e^t dt + c = e^t(t-1)+c$

$= x\tan^{-1}y = e^{\tan^{-1}}y(\tan^{-1}y-1)+c$

9. $(y+x)\dfrac{dy}{dx} = y-x \Rightarrow \dfrac{dy}{dx} = \dfrac{y-x}{y+x}$

Let $y = vx$

$\Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\Rightarrow v + x\dfrac{dv}{dx} = \dfrac{vx-x}{vx+x} = \dfrac{v-1}{v+1}$

$\Rightarrow x\dfrac{dv}{dx} = \dfrac{v-1}{v+1} - v = \dfrac{v-1-v^2-v}{v+1}$

$\Rightarrow \int\dfrac{dx}{x} = -\int dv\dfrac{(1+v)}{1+v^2}$

$\Rightarrow \log x = -\left[\int\dfrac{1}{1+v^2}dv + \int\dfrac{vdv}{1+v^2}\right]$

$\Rightarrow \log x = -\left[\tan^{-1}v + \dfrac{1}{2}\int\dfrac{2v}{1+v^2}dv\right]+c$

Let $1+v^2 = t$

$2vdv = dt$

$\Rightarrow \log x = -\left[\tan^{-1}v + \dfrac{1}{2}\int\dfrac{dt}{t}\right]+c$

$\Rightarrow \log x = -\left[\tan^{-1}v + \dfrac{\log t}{2}\right]+c$

$\Rightarrow \log x = -\left[\tan^{-1}\dfrac{y}{x} + \dfrac{\log\frac{y}{x}}{2}\right]+C$

10. Let, $I = \int\sqrt{1-\sin 2x}\,dx$

$= \int\sqrt{\sin^2 x+\cos^2 x-2\sin x.\cos x}\,dx$

$= \int\sqrt{(\sin x-\cos x)^2}\,dx$

$= \int(\sin x-\cos x)dx$

$= \int\sin x\,dx - \int\cos x\,dx$

$= -\cos x - \sin x + c$; **Ans.**

11. Let, $I = \int \sin 4x \cos 3x\, dx = \frac{1}{2}\int 2\sin 4x.\cos 3x\, dx$

[$\because$ $2\sin A\cos B = \sin(A+B) + \sin(A-B)$]

$$\Rightarrow\ I = \frac{1}{2}\int [\sin(4x+3x) + \sin(4x-3x)]\, dx$$

$$= \frac{1}{2}\int (\sin 7x + \sin x) dx$$

$$= \frac{1}{2}\left[-\frac{\cos 7x}{7} + \cos x\right] + C$$

$$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C\text{; } \textbf{Ans.}$$

12. Let, $I = \int \frac{\operatorname{cosec} x}{\operatorname{cosec} x + \cot x} dx$

$$= \int \frac{\frac{1}{\sin x}}{\frac{1}{\sin x} + \frac{\cos x}{\sin x}} dx = \int \frac{\frac{1}{\sin x}}{\frac{1+\cos x}{\sin x}} dx$$

$$= \int \frac{1}{1+\cos x} dx = \int \frac{1}{2\cos^2 x/2} dx$$

$$= \frac{1}{2}\int \sec^2 x/2\, dx = \frac{1}{2}\frac{\tan x/2}{\frac{1}{2}} + k$$

$$= \frac{1}{2} \times \frac{2}{1}\tan\frac{x}{2} + k = \tan\frac{x}{2} + k\text{; } \textbf{Ans.}$$

13. $\int \tan^3 x\, dx = \int \tan x \,.\, \tan^2 x\, dx$

$$= \int \tan x (\sec^2 x - 1)\, dx$$

$$= \int \tan x \sec^2 x\, dx - \int \tan x\, dx$$

Now let $\tan x = y \Rightarrow \sec^2 x\, dx = dy$

$$\therefore\ \int y\, dy - \int \tan x\, dx = \frac{y^2}{2} - \log(\sec x) + C$$

$$= \frac{\tan^2 x}{2} - \log(\sec x) + C$$

Ans.

4. $\int \sin^2 x \cdot \cos^2 x dx$

$$= \frac{1}{4}\int 4\sin^2 x \cos^2 x dx$$

$$= \frac{1}{4}\int (2\sin x \cos x)^2 dx$$

$$= \frac{1}{4}\int (\sin 2x)^2 \cdot dx = \frac{1}{4}\int \sin^2 2x dx$$

$$= \frac{1}{4}\int \frac{1-\cos 4x}{2} dx$$

$$= \frac{1}{8}\int (1-\cos 4x) dx = \frac{1}{8}\left\{\int dx - \int \cos 4x\, dx\right\}$$

$$= \frac{1}{8}\left\{x - \frac{\sin 4x}{4}\right\} + C = \frac{x}{8} - \frac{\cos 4x}{32} + C\ \textbf{Ans.}$$

15. $\int_0^{\pi/2} \sin^4 x\, dx = \frac{1}{4}\int_0^{\pi/2} (1-\cos 2x)^2 dx$

$$= \frac{1}{4}\cdot\int_0^{\pi/2} (1 - 2\cos 2x + \cos^2 2x)\, dx$$

$$= \frac{1}{4}\cdot\int_0^{\pi/2} \left[1 - 2\cos 2x + \frac{(1+\cos 4x)}{2}\right] dx$$

$$= \frac{1}{4}\cdot\int_0^{\pi/2} \left(\frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x\right) dx$$

$$= \frac{3}{8}\cdot\int_0^{\pi/2} dx - \frac{1}{2}\int_0^{\pi/2} \cos 2x dx + \frac{1}{8}\int_0^{\pi/2} \cos 4x\, dx$$

$$= \frac{3}{8}\cdot[x]_0^{\pi/2} - \frac{1}{2}\cdot\left[\frac{\sin 2x}{2}\right]_0^{\pi/2} + \frac{1}{8}\cdot\left[\frac{\sin 4x}{4}\right]_0^{\pi/2}$$

$$= \left(\frac{3\pi}{16} - 0 + 0\right) = \frac{3\pi}{16}\ \textbf{Ans.}$$

16. Let $z = 1 + \sin x$, then $dz = \cos x\, dx$

When $x = 0$, $z = 1 + \sin 0 = 1 + 0 = 1$

and when $x = \frac{\pi}{2}$, $z = 1 + \sin\frac{\pi}{2} = 1 + 1 = 2$

Now, $I = \int_0^{\pi/2} \frac{\cos x}{(1+\sin x)^2} dx = \int_1^2 \frac{dz}{z^2} = \int_1^2 z^{-2} dz$

$$= \left[\frac{z^{-1}}{-1}\right]_1^2 = -\left[\frac{1}{z}\right]_1^2 = -\left[\frac{1}{2} - 1\right] = \frac{1}{2}\ \textbf{Ans.}$$

17. Let $z = \log_e x$, then $dz = \frac{1}{x} dz$

Again when $x = 1$, $z = \log_e 1 = 0$

and when $x = e$, $z = \log_e e = 1$

Now, $I = \int_1^e \frac{\cos(\log_e x)}{x} dx = \int_0^1 \cos z dz = [\sin z]_0^1$

$= \sin 1 - \sin 0 = \sin 1$; **Ans.**

18. Here $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$$\vec{a} \times \vec{b} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= \hat{i}(-14+14) - \hat{j}(2-21) + \hat{k}(-2+21)$$

$$= 19\hat{i} + 19\hat{j}$$

$$\therefore\ |\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2}$$

$$= \sqrt{(19)^2(1+1)} = 19\sqrt{2}$$

19. Let the angle to be θ.

Then, $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$$\therefore\ \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

$$= \frac{(\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i}+\hat{j}-\hat{k})}{|\hat{i}+\hat{j}+\hat{k}||\hat{i}+\hat{j}-\hat{k}|}$$

$$= \frac{(1+1-1)}{[\sqrt{1^2+1^2+1^2}][\sqrt{1^2+1^2+(-1)^2}]}$$

$$= \frac{1}{\sqrt{3}\cdot\sqrt{3}} = \frac{1}{3}$$

$\therefore \quad \theta = \cos^{-1}\left(\frac{1}{3}\right)$ **Ans.**

20. Direction ratio of the given lines and those of normal to the given plane are 3, 2, 4, and 2, 1, –3.

$\therefore$ Acute angle θ between the given line and plane is given by

$$\sin\theta = \left|\frac{3\times 2+2\times 1+4\times(-3)}{\sqrt{9+4+16}\sqrt{4+1+9}}\right| = \frac{4}{\sqrt{406}}$$

Hence $\theta = \sin^{-1}\left(\frac{4}{\sqrt{406}}\right)$

21. We know that

$$P(A\cup B) = P(A)+P(B)-P(A\cap B)$$
$$= P(A)+P(B)-P(A)\cdot P(B)$$

[Here, $P(A\cap B) = P(A)\cdot P(B)$ as the events A and B are independent]

$\Rightarrow \quad P(A\cup B) = P(A)+P(B)[1-P(A)]$ ...(1)

Now $\quad P(A\cup B) = 0.5,\ P(A) = 0.2$

$\therefore$ From (1),

$$0.5 = 0.2+P(B)[1-0.2]$$

$\Rightarrow \quad 0.3 = P(B)\times 0.8$

$\Rightarrow \quad P(B) = \frac{0.3}{0.8} = \frac{3}{8} = 0.375$ **Ans.**

22. Let, $\operatorname{cosec}^{-1}x = \theta$, then $\operatorname{cosec}\theta = x$

But $\operatorname{cosec}\theta = \sec\left(\frac{\pi}{2}-\theta\right)$

$\therefore \quad \sec\left(\frac{\pi}{2}-\theta\right) = x \quad \Rightarrow \sec^{-1}x = \frac{\pi}{2}-\theta$

$\Rightarrow \quad \theta+\sec^{-1}x = \frac{\pi}{2}$

$\therefore \quad \operatorname{cosec}^{-1}x+\sec^{-1}x = \frac{\pi}{2}$ **Proved**

23. $\because$ cos (A – B) = cos A cos B + sin A sin B

$= \cos A\cos B + \sqrt{(1-\cos^2 A)(1-\cos^2 B)}$

Let, $\quad \cos A = x \Rightarrow A = \cos^{-1}x$ and
$\cos B = y \Rightarrow B = \cos^{-1}y$

$\therefore \quad \cos(A-B) = xy+\sqrt{(1-x^2)(1-y^2)}$

$\Rightarrow \quad A-B = \cos^{-1}\left|xy+\sqrt{(1-x^2)(1-y^2)}\right|$

$\therefore \quad \cos^{-1}x-\cos^{-1}y = \cos^{-1}\left[xy+\sqrt{(1-x^2)(1-y^2)}\right]$.

Proved

24. $\tan\left(\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2}+\frac{1}{2}\cos^{-1}\frac{1-x^2}{1+x^2}\right)$

putting $x = \tan\theta$, we get

$$\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)\right\}$$

$$= \tan\left\{\frac{1}{2}\sin^{-1}(\sin 2\theta)+\frac{1}{2}\cos^{-1}(\cos 2\theta)\right\}$$

$$= \tan\left\{\left(\frac{1}{2}\times 2\theta\right)+\left(\frac{1}{2}\times 2\theta\right)\right\}$$

$$= \tan(\theta+\theta) = \tan 2\theta$$

$$= \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2x}{1-x^2} = \text{R.H.S.}$$

25. $\tan^{-1}(\sqrt{3})-\cot^{-1}(-\sqrt{3})$

$$= \tan^{-1}(\sqrt{3})+\cot^{-1}(+\sqrt{3}) = \frac{\pi}{3}+\frac{\pi}{6} = \frac{\pi}{2}$$

26. $y = \sin(2\sin^{-1}x)$

Now $\frac{dy}{dy} = \frac{d\sin(2\sin^{-1}x)}{dx}$

$$= \frac{d\sin(2\sin^{-1}x)}{d(2\sin^{-1}x)}\cdot\frac{d(2\sin^{-1}x)}{dx}$$

$$= \cos(2\sin^{-1}x)\cdot 2\cdot\frac{1}{\sqrt{1-x^2}}$$

$$= \frac{2}{\sqrt{1-x^2}}\cos(2\sin^{-1}x)$$ **Ans.**

27. $A = \begin{bmatrix} 1 & 1 \\ x & y \\ x^2 & y^2 \end{bmatrix}$

then, $A' = \begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \end{bmatrix}$

$\therefore \quad AA' = \begin{bmatrix} 1 & 1 \\ x & y \\ x^2 & y^2 \end{bmatrix}\begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \end{bmatrix}$

$$= \begin{bmatrix} 1\cdot 1+1\cdot 1 & 1\cdot x+1\cdot y & 1\cdot x^2+1\cdot y^2 \\ x\cdot 1+y\cdot 1 & x\cdot x+y\cdot y & x.x^2+y\cdot y^2 \\ x^2\cdot 1+y^2\cdot 1 & x^2\cdot x+y^2\cdot y & x^2\cdot x^2+y^2\cdot y^2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & x+y & x^2+y^2 \\ x+y & x^2+y^2 & x^3+y^3 \\ x^2+y^2 & x^3+y^3 & x^4+y^4 \end{bmatrix}$$ **Ans.**

28. $x = a\cos^2\theta$

$$\frac{dx}{d\theta} = a.\ 2\cos\theta.\ \frac{d}{d\theta}\cos\theta$$

$$\Rightarrow \frac{dx}{d\theta} = -a{\cdot}2\cos\theta{\cdot}\sin\theta$$

$$\Rightarrow \frac{dx}{d\theta} = -a\sin 2\theta \quad \text{...(i)}$$

$y = a\sin^2\theta$

$$\Rightarrow \frac{dy}{d\theta} = a{\cdot}2\sin\theta\ \frac{d}{d\theta}\sin\theta$$

$$\Rightarrow \frac{dy}{d\theta} = a{\cdot}2\sin\theta{\cdot}\cos\theta$$

$$\Rightarrow \frac{dy}{d\theta} = a{\cdot}\sin 2\theta \quad \text{...(ii)}$$

From eq. (ii) ÷ eq. (i), we have

$$\frac{dy/d\theta}{dx/d\theta} = \frac{a\sin 2\theta}{-a\sin 2\theta}$$

$$\therefore \frac{dy}{dx} = -1 \textbf{ Ans.}$$

29. $x^2 + y^2 = 3$ (straight line)

$$\Rightarrow 2x + 2y{\cdot}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{\sqrt{2}}$$

$$\therefore M = -\frac{1}{\sqrt{2}} \textbf{ Ans.}$$

30. L.H.S. $= (\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})$

$= \vec{a}\cdot(\vec{a}-\vec{b})+\vec{b}\cdot(\vec{a}-\vec{b})$

$= \vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}-\vec{b}\cdot\vec{b}$

$= \vec{a}^2-\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{b}-\vec{b}^2$ $\quad (\because \vec{b}\cdot\vec{a} = \vec{a}\cdot\vec{b})$

$= a^2 - b^2$

= R.H.S. **Proved.**

31. Given : $\sin y = x\sin(a+y)$ ... (1)

Diff. w.r.t. y

$$\cos y = \frac{dx}{dy}\sin(a+y) + x\cos(a+y)$$

$$\Rightarrow \frac{dx}{dy}.\sin(a+y) = \cos y - x\cos(a+y)$$

$$= \cos y - \frac{\sin y}{\sin(a+y)}.\cos(a+y) \quad \text{[Use (1)]}$$

$$= \frac{\sin(a+y)\cos y - \sin y\cos(a+y)}{\sin(a+y)}$$

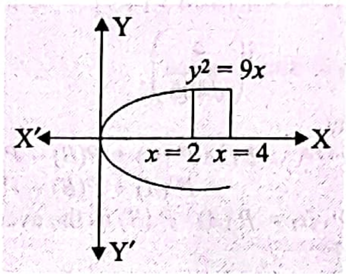
$$= \frac{\sin(a+y-y)}{\sin(a+y)} = \frac{\sin a}{\sin(a+y)}$$

$$\Rightarrow \frac{dy}{dx}.\frac{1}{\sin(a+y)} = \frac{\sin(a+y)}{\sin a}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

32. Since, the given curve $y^2 = 9x$ is a parabola which is symmetrical about X-axis ($\because$ the power of y is even) and passes through theorigin.

The area of the region bounded by the curve, $y^2 = 9x$, $x = 2$ and $xd = 4$ and the X-axis is the area shown in the figure.



Required area (shaded region)

$$= \int_2^4 |y|\,dx = \int_2^4 3\sqrt{x}\,dx$$

$$= 2\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_2^4 = \frac{3\times 2}{3}[4^{3/2} - 2^{3/2}]$$

$$= 2\,[4\sqrt{4} - 2\sqrt{2}] = 2\,[8 - 2\sqrt{2}] = 4\,[4 - 4\sqrt{2}] \text{ sq. unit.}$$

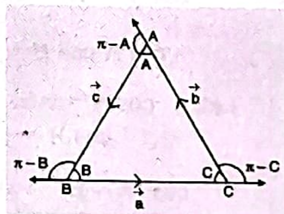
33. In triangle ABC

Let $\vec{BC} = \vec{a}, \vec{CA} = \vec{b}, \vec{AB} = \vec{c}$

then $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ ...(1)

Taking the vector product of both sides of

(1) with $\vec{a}$, we have



$\vec{a}\times(\vec{a}+\vec{b}+\vec{c}) = \vec{a}\times\vec{0}$

or $\vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c} = \vec{0}$

or $\vec{a}\times\vec{b}+\vec{a}\times\vec{c} = \vec{0}$ $\quad [\because \vec{a}\times\vec{a} = \vec{0}]$

or $\vec{a}\times\vec{b}-\vec{c}\times\vec{a} = \vec{0}$

or $\vec{a}\times\vec{b} = \vec{c}\times\vec{a}$ ...(2)

Similarly, taking the vector product of both sides of (1) with $\vec{b}$, we have $\vec{a} \times \vec{b} = \vec{b} \times \vec{c}$ ...(3)

From (2) and (3), we get $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

$$\Rightarrow \quad |\vec{a} \times \vec{b}| = |\vec{b} \times \vec{c}| = |\vec{c} \times \vec{a}|$$

or $\quad ab \sin(\pi - C) = bc \sin(\pi - A) = ca \sin(\pi - B)$

or $\quad ab \sin C = bc \sin A = ca \sin B$

Dividing throught by abc, we get

$$\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b} \text{ or, } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ **Proved.**

34. Given line $\dfrac{x-2}{6} = \dfrac{y-1}{\lambda} = \dfrac{x+5}{-4}$ is perpendicular to the plane $3x - y - 2z = 7$

$$\therefore \frac{6}{3} = \frac{\lambda}{-1} = \frac{-4}{-2}$$

$$\Rightarrow \quad 2 = \frac{\lambda}{-1} = 2 \quad \therefore \quad \lambda = -2$$

35. $\quad P(A) = \dfrac{1}{4}, P(B) = \dfrac{1}{3}$

and $P(A \cup B) = \dfrac{1}{2}$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{4} + \frac{1}{3} - \frac{1}{2}$$

$$\Rightarrow P(A \cap B) = \frac{3+4-6}{12} = \frac{1}{12} \quad \text{... (i)}$$

$$P(A) \times P(B) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12} \quad \text{... (ii)}$$

From equation (i) and (ii)

Here $P(A \cap B) = P(A) \times P(B)$.

So A and B are independent events.

36. Let $\Delta = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$

Taking bc common from R_1, ca common from R_2 and ab common from R_3, we have

$$\Delta = a^2b^2c^2 \begin{vmatrix} bc & 1 & \frac{1}{b}+\frac{1}{c} \\ ca & 1 & \frac{1}{c}+\frac{1}{a} \\ ab & 1 & \frac{1}{a}+\frac{1}{b} \end{vmatrix}$$

$$= a^2b^2c^2 \begin{vmatrix} bc & 1 & \frac{1}{b}+\frac{1}{c} \\ c(a-b) & 0 & \frac{1}{a}-\frac{1}{b} \\ b(a-c) & 0 & \frac{1}{a}-\frac{1}{c} \end{vmatrix}$$

$[R_2 \to R_2 - R_1, R_3 \to R_3 - R_1]$

$$= a^2b^2c^2 (a-b)(a-c) \begin{vmatrix} bc & 1 & \frac{1}{b}+\frac{1}{c} \\ c & 0 & -\frac{1}{ab} \\ b & 0 & -\frac{1}{ac} \end{vmatrix}$$

$$= \frac{a^2b^2c^2(a-b)(a-c)}{ab.ac} \begin{vmatrix} bc & 1 & \frac{1}{b}+\frac{1}{c} \\ abc & 0 & -1 \\ abc & 0 & -1 \end{vmatrix}$$

[Multiply R_2 by ab and R_2 by ac]

[$\because R_2$ and R_3 are identical rows]

$= bc\,(a-b)\,(a-c) \times 0 = 0$; **Proved.**

37. $\quad 3x + 5y \leq 15 \qquad 5x + 2y = 10$

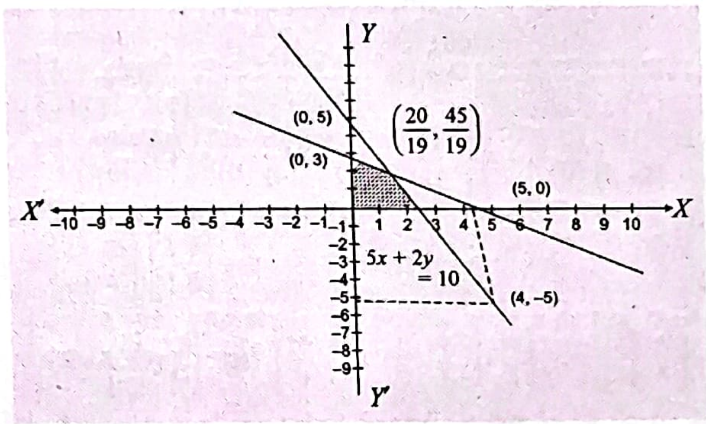
$3x + 5y = 15 \qquad \Rightarrow 2y = 10 - 5x$

$\Rightarrow 5y = 15 - 3x \qquad \Rightarrow y = \dfrac{10-5x}{2}$

$y = \dfrac{15-3x}{5}$

x	0	5	–5
y	3	0	6

x	2	4	0
y	0	–5	5



Feasible solution

$$= (2, 0), \left(\frac{20}{19}, \frac{45}{19}\right), (0, 3)$$

at (2.0)

$z = 5x + 3y = 5 \times 2 + 3 \times 0 = 10$

at $\left(\frac{20}{19}, \frac{45}{19}\right)$

$$z = 5x + 3y = 5\times\frac{20}{19}+3\times\frac{45}{19} = \frac{100}{19}+\frac{135}{19} = \frac{235}{19} = 12.37$$

at (0, 3)

$z = 5x + 3y = 5 \times 0 + 3 \times 3 = 9$

$z_{max} = \frac{235}{19}$ or 12.37 **Ans.**

38. Let $I = \int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x})\, dx$

Put : $\sqrt{\tan x} = t$

$\Rightarrow \tan x = t^2$

$\Rightarrow \sec^2 x\, dx = 2t\, dt$

$\Rightarrow dx = \frac{2t dt}{1+t^4}$

Also $\sqrt{\cot x} = \frac{1}{t}$

$x = 0 \Rightarrow t = 0; \quad x = \frac{\pi}{2} \Rightarrow t = \infty$

$$\therefore \quad I = \int_0^{\infty}\left[t+\frac{1}{t}\right]\cdot\frac{2t\, dt}{1+t^4}$$

$$= \int_0^{\infty}\frac{t^2+1}{t^4+1}dt$$

$$= \int_0^{\infty}\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt$$

Put : $t - \frac{1}{t} = y \Rightarrow \left(1+\frac{1}{t^2}\right)dt = dy$

Now, $\int\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt = \int\frac{dy}{y^2+2}$

$$= \int\frac{dy}{y^2+(\sqrt{2})^2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}t}\right)$$

$$\therefore \quad I = \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\frac{t^2-2}{\sqrt{2}t}\right)\right]_0^{\infty}$$

$$= \frac{1}{\sqrt{2}}[\tan^{-1}\infty - \tan^{-1}(-\infty)]$$

$$= \frac{1}{\sqrt{2}}\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right] = \frac{\pi}{\sqrt{2}}$$ **Ans.**

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