# MODEL PAPER – 4

**Time : 3 Hours + 15 Minutes ]** **[ Total Marks : 100**

**Instructions to the Candidates :**

1. Candidates are required to give their answers in their own words as far as practicable.
2. Figures in the right hand margin indicate full marks.
3. 15 Minutes of extra time has been allotted for the candidates to read the questions carefully.
4. This question paper is divided into two sections—**SECTION – A** and **SECTION – B**.
5. In **SECTION – A** there are **100 Objective Type Question,** out of which only 50 objective question be answered. Darken the circle with blue/black ball pen against the correct option on OMR Sheet provided to you. Do not use **Whitener/Liquid/ Blade/Nail** on OMR paper; otherwise the result will be invalid.
6. In **SECTION – B**, there are **30 Short Answer Type Questions** (each carrying 2 marks), out of which any 15 questions are be answered.

   Apart from this, there are **8 Long Answer Type Question** (Each Carrying 5 marks), out of which 4 Questions are to be answered.
7. Use of any electronic device is prohibited.

## SECTION – A : Objective Type Questions

**Direction :** *There are 100 Objective Type Questions, out of which only 50 objective questions to be answered. Mark the correct option on the* **OMR** *Answer Sheet.* $50 \times 1 = 50$

**1.** $\tan^{-1}\left(\frac{x+y}{1-xy}\right) =$

(A) $\sin^{-1}(x+y)$ (B) $\cos^{-1}(x+y)$

(C) $\tan^{-1}(x+y)$ (D) $\tan^{-1}x+\tan^{-1}y$

**2.** $\frac{d}{dx}(e^{-3x}) =$

(A) $\frac{e^{-3x}}{3}$ (B) $\frac{e^{-3x}}{-3}$

(C) $3e^{-3x}$ (D) $-3e^{-3x}$

**3.** $\frac{d}{dx}(11^x) =$

(A) $x11^{x-1}$ (B) $11^x.\log x$

(C) $11^x.\log 11$ (D) $\frac{11^x}{\log 11}$

**4.** $3\int_0^3 x^3 dx =$

(A) $\frac{81}{4}$ (B) $\frac{243}{4}$ (C) 0 (D) $\frac{9}{4}$

**5.** $\int_{-1}^{1} \sin^{17} x \cos^3 x \, dx =$

(A) $\frac{12}{5}$ (B) 0 (C) 1 (D) $\frac{3}{5}$

**6.** The solution of the differential equation $dx + dy = 0$ is

(A) $x = ky$ (B) $x^2 + y^2 = k$

(C) $x + y = k$ (D) $xy = k$

**7.** $x \in R, \operatorname{cosec}(\tan^{-1}x+\cot^{-1}x) =$

(A) 0 (B) 1 (C) $\frac{2}{\sqrt{3}}$ (D) 2

**8.** The maximum value $Z = x - 3y$ subject to constraints $x+y \le 13,\ x \ge 0,\ y \ge 0$ is :

(A) 39 (B) 26

(C) 13 (D) –26

**9.** Direction ratios of the normal to the plane $x + 2y - 3z + 15 = 0$ are :

(A) 1, 2, 3 (B) 1, –2, 3

(C) 1, 2, –3 (D) 1, 2, 15

**10.** $\vec{i}.\vec{i} =$

(A) 0 (B) 1

(C) –1 (D) $\vec{j}$

**11.** $(11\vec{i}-7\vec{j}-\vec{k}).(8\vec{i}-\vec{j}-5\vec{k}) =$

(A) 95 (B) 100

(C) 400 (D) 88

**12.** $[-1]\ [1\ \ -1] =$

(A) [0] (B) [–1 1] (C) $\begin{bmatrix}-1\\1\end{bmatrix}$ (D) [2 –2]

**13.** $\begin{bmatrix}13 & 15\\-1 & 4\end{bmatrix}\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix} =$

(A) $\begin{bmatrix}13 & 15\\-1 & 4\end{bmatrix}$ (B) $\begin{bmatrix}15 & 0\\0 & 8\end{bmatrix}$ (C) $\begin{bmatrix}26 & 30\\-2 & 8\end{bmatrix}$ (D) $\begin{bmatrix}13 & 0\\0 & 6\end{bmatrix}$

**14.** If $A = \{a, b, c\}$, $B = \{1, 2, 3\}$ and $f = \{(a, 1), (b, 2), (c, 2)\}$ then what type of a function is $f$ ?

(A) one-one into (B) many-one into

(C) many-one onto (D) one-one onto

15. Principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ :

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) None

16. $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\pi$

17. If $y = \sin x^2$ then $\frac{dy}{dx}=$

(A) $2x \sin x^2$ (B) $x \sin x$
(C) $x \cos x^2$ (D) $2x \cos x^2$

18. $\int_0^{\pi/2} \sin^2 x dx =$

(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $-\frac{\pi}{2}$

19. $\int_0^{\pi/2} e^x(\sin x+\cos x)\, dx =$

(A) $e^{\frac{\pi}{2}}$ (B) 1 (C) $e^{\frac{\pi}{4}}$ (D) $e^x$

20. The direction ratios of a line are 1, 3, 5 then its direction cosines are :

(A) $\frac{1}{\sqrt{35}},\frac{3}{\sqrt{35}},\frac{5}{\sqrt{35}}$ (B) $\frac{1}{9},\frac{1}{3},\frac{5}{9}$

(C) $\frac{5}{\sqrt{35}},\frac{3}{\sqrt{35}},\frac{1}{\sqrt{35}}$ (D) None

21. The direction ratios of the normal to the plane $7x + 4y - 2z + 5 = 0$ are :

(A) 7, 4, –2 (B) 7, 4, 5
(C) 7, 4, 2 (D) 4, –2, 5

22. If $\begin{bmatrix}1-x & 2\\ 18 & 6\end{bmatrix}=\begin{bmatrix}6 & 2\\ 18 & 6\end{bmatrix}$, then $x =$

(A) ± 6 (B) 6 (C) –5 (D) 7

23. Let $A$ be a non-singular matrix of the order 2 × 2 then $|A^{-1}| =$

(A) $|A|$ (B) $\frac{1}{|A|}$ (C) 0 (D) 1

24. If $P(A)=\frac{3}{8}, P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{4}$, then $P\left(\frac{A}{B}\right)=$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{8}$

25. If $A$ and $B$ are two independent events, then :

(A) $P(AB') = P(A)P(B)$ (B) $P(AB') = P(A)P(B')$
(C) $P(AB') = P(A') + P(B)$ (D) $P(AB') = P(A)P(B)$

26. $\int \frac{2}{x} dx =$

(A) $k + 2x$ (B) $k-\frac{2}{x^2}$

(C) $k - 2x$ (D) $k + 2 \log |x|$

27. $\int_0^1 x^2 dx =$

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) 1

28. If $y = x \tan y$, then $\frac{dy}{dx}=$

(A) $\frac{\tan x}{x-x^2-y^2}$ (B) $\frac{y}{x-x^2-y^2}$

(C) $\frac{\tan y}{y-x}$ (D) $\frac{\tan x}{x-y^2}$

29. $\frac{d}{dx}[\log x] = ?$

(A) $\frac{1}{x}$ (B) $-\frac{1}{x^2}$ (C) 1 (D) $\frac{1}{x^2}$

30. $\cos^{-1}(2x - 1) =$

(A) $2 \cos^{-1} x$ (B) $\cos^{-1}\sqrt{x}$
(C) $2 \cos^{-1}\sqrt{x}$ (D) None of these

31. $2 \cot^{-1} 3 + \cot^{-1} 7 =$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\pi$ (D) $\frac{\pi}{6}$

32. The solution of the differentiation equation $(x + 4)(dx - dy) = dx + dy$ is :

(A) $x - y = \log(x + y) + C$ (B) $x + y = \log(x - y) + C$
(C) $x^2 + y^2 = x + y + C$ (D) $x^2 - y^2 = x + y + C$

33. The order of the differentiation equation $\frac{d^2y}{dx^2}+x^3\left(\frac{dy}{dx}\right)^3=x^4$ is :

(A) 1 (B) 2 (C) 4 (D) 3

34. If $P(A)=\frac{6}{11}, P(B)=\frac{5}{11}, P(A\cup B)=\frac{7}{11}$, then $P(A \cap B) = ?$

(A) $\frac{4}{11}$ (B) $\frac{5}{11}$ (C) $\frac{7}{11}$ (D) $\frac{9}{11}$

35. If $P(A)=\frac{3}{8}$, $P(B)=\frac{1}{2}$, $P(A\cap B)=\frac{1}{4}$ then $P(A\cup B)=$

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$

36. If the planes $a_1x + b, y + c, z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular to each other then :

(A) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ (B) $\frac{a_1}{a_2}+\frac{b_1}{b_2}+\frac{c_1}{c_2}=0$

(C) $a_1a_2+b_1b_2+c_1c_2=0$ (D) $a_1^2a_2^2+b_1^2b_2^2+c_1^2c_2^2=0$

37. The scalar product of $5\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}-4\hat{j}+7\hat{k}$ is :

(A) 10 (B) –10 (C) 15 (D) –15

38. $\int\frac{-1}{1+x^2}dx=$

(A) $\tan^{-1} x + k$ (B) $\sec^{-1} x + k$
(C) $\operatorname{cosec}^{-1} x + k$ (D) $\cot^{-1} x + k$

39. $\int \frac{dx}{x^2+a^2} =$

(A) $\frac{1}{a}\tan^{-1}\frac{x}{a}+k$ (B) $\frac{1}{a}\tan^{-1}(x+a)+k$

(C) $\sin^{-1}\frac{x}{a}+k$ (D) $\cos^{-1}\frac{x}{a}+k$

40. The value of the determinant $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 7 & 9 \\ 4 & 8 & 16 \end{bmatrix}$ is :

(A) 23 (B) 0

(C) 1 (D) None of these

41. The inverse of $A = \begin{vmatrix} 2 & 3 \\ 5 & k \end{vmatrix}$ will not be obtained if $k$ has the value :

(A) 1 (B) 3/2 (C) 5/2 (D) 15/2

42. $\frac{d}{dx}(\sin\sqrt{x}) =$

(A) $\cos\sqrt{x}$ (B) $\frac{\cos\sqrt{x}}{\sqrt{x}}$

(C) $\frac{1}{\sqrt{x}}\cdot\cos\sqrt{x}$ (D) $\frac{1}{2\sqrt{x}}\cdot\cos\sqrt{x}$

43. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then AI = ?

(A) $\begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ (D) None

44. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} =$

(A) $\begin{bmatrix} a & 2b \\ 3c & 4d \end{bmatrix}$ (B) $\begin{bmatrix} 1+a & 2b \\ 3+c & 4d \end{bmatrix}$

(C) $\begin{bmatrix} 1+a & 2+b \\ 3+c & 4+d \end{bmatrix}$ (D) $\begin{bmatrix} 1+a & 2+b \\ 3c & 4+d \end{bmatrix}$

45. $\tan^{-1} 1 =$

(A) $\pi$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

46. $\cos(\sec^{-1} x + \operatorname{cosec}^{-1} x) =$

(A) 1 (B) –1 (C) 0 (D) $\frac{1}{\sqrt{2}}$

47. $\frac{d}{dx}(\cot^{-1} x) =$

(A) $\frac{1}{1+x^2}$ (B) $\frac{1}{1+x}$ (C) $-\frac{1}{1+x}$ (D) $-\frac{1}{1+x^2}$

48. $\frac{d}{dx}[\log(\sec x + \tan x)] =$

(A) $\frac{1}{\sec x + \tan x}$ (B) $\sec x$

(C) $\tan x$ (D) $\sec x + \tan$

49. If $\alpha$, $\beta$, $\gamma$ are the angle which a half ray makes with the positive directions of the axes then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

(A) 1 (B) 2 (C) 0 (D) –1

50. The distance of the point (3, 4, 5) from $x$-axis is :

(A) 3 (B) 5

(C) $\sqrt{41}$ (D) None of these

51. If $f(-x) = -f(x)$ then $\int_{-a}^{a} f(x)dx =$

(A) $2\int_{-a}^{a} f(x)dx$ (B) 0

(C) 1 (D) –1

52. $\int_{\alpha}^{\beta} \varphi(x)\,dx + \int_{\beta}^{\alpha} \varphi(x)\,dx =$

(A) 1 (B) $2\int_{\alpha}^{\beta} \varphi(x)\,dx$

(C) $-2\int_{\beta}^{\alpha} \varphi(x)\,dx$ (D) 0

53. If $\vec{a} = 2\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{b} = -\vec{i} + \vec{j} + \vec{k}$ then :

(A) $\vec{a}\cdot\vec{b} = 5$ (B) $\vec{a}\cdot\vec{b} = -5$

(C) $\vec{a}\cdot\vec{b} = 0$ (D) None of these

54. The number of unit vector (s) perpendicular to a plane is :

(A) 1 (B) 2

(C) 3 (D) infinite

55. Solution of the differential equation $xdy + ydx = 0$ is :

(A) $\frac{x^2}{2}+\frac{y^2}{2}=k$ (B) $\frac{x^2}{2}-\frac{y^2}{2}=k$

(C) $xy - k$ (D) None of these

56. The radius of a circle is increasing at the rate of 0.4 cm/s. The rate of increase of its circumference is :

(A) 0.4 $\pi$ cm/s (B) 0.8 $\pi$ cm/s

(C) 0.8 cm/s (D) None of these

57. $\frac{d}{dx}(2\tan^{-1} x) =$

(A) $\frac{1}{1+x^2}$ (B) $\frac{2}{1+x^2}$

(C) $\frac{1}{2}\cdot\frac{1}{(1+x^2)}$ (D) $\frac{1}{2}\cdot\frac{1}{(1-x^2)}$

58. If $A$ is an invertible matrix of order 2 then det $(A^{-1})$ is equal to :

(A) det $(A)$ (B) $\frac{1}{\det(A)}$

(C) 1 (D) 0

59. The value of the determinant $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$ is :

(A) 0 (B) 1

(C) –1 (D) $a+b+c$

60. If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{3}$ and $P(A\cap B) = \frac{1}{4}$, then $P(A'\cap B') =$

(A) $\frac{13}{8}$ (B) $\frac{13}{4}$ (C) $\frac{13}{24}$ (D) $\frac{13}{9}$

61. If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A\cap B) = \frac{1}{4}$ then $P\left(\frac{A'}{B'}\right) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) $\frac{3}{8}$

62. $\vec{k}\cdot\vec{i} =$

(A) 0 (B) 1 (C) $\vec{k}$ (D) $\vec{i}$

63. If $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}, \vec{b} = \vec{i} + 3\vec{j} - 2\vec{k}$ then $\vec{a}\cdot\vec{b} =$:

(A) 1 (B) 20 (C) 30 (D) –30

64. $\int_2^1 \frac{dx}{x} = ?$

(A) $\log\frac{2}{3}$ (B) $\log\frac{3}{2}$ (C) $\log\frac{1}{2}$ (D) $\log\frac{x}{2}$

65. $\int (x+2)\,dx =$

(A) $(x+2)^3 + k$ (B) $\frac{x^2}{2} + k$

(C) $\frac{x^2}{2} + 2x + k$ (D) $\log(x+2) + k$

66. $f : A \to B$ will be an onto function of :

(A) $f(A) \subset B$ (B) $f(A) = B$

(C) $f(A) \supset B$ (D) $f(A) \neq B$

67. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to :

(A) $\pi$ (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

68. $\cos^{-1}\frac{1-x^2}{1+x^2} =$ ...., $(|x| \le 1)$

(A) $2\cos^{-1}x$ (B) $2\sin^{-1}x$

(C) $2\tan^{-1}x$ (D) $\tan^{-1}2x$

69. $\vec{a}.\vec{a} =$

(A) 0 (B) 1 (C) $|\vec{a}|^2$ (D) $|\vec{a}|$

70. The projection of the vector $2\hat{i} - \hat{j} + \hat{k}$ on the vector $\hat{i} - 2\hat{j} + \hat{k}$ is :

(A) $\frac{4}{\sqrt{6}}$ (B) $\frac{5}{\sqrt{6}}$ (C) $\frac{4}{\sqrt{3}}$ (D) $\frac{7}{\sqrt{6}}$

71. $\int x^n dx, n \neq 0 =$

(A) $\frac{x^{n-1}}{n-1} + C$ (B) $\frac{x^{n+1}}{n+1} + C$

(C) $x^{n+1} + C$ (D) $x^{n-1} + C$

72. $\int \sec x\,dx =$

(A) $\log\sec x$ (B) $\log\sec x + \tan x$

(C) $\log(\sec x - \tan x)$ (D) $\sec x \cdot \tan x$

73. $\frac{d}{dx}(\sqrt{\tan x}) = ?$

(A) $2\sqrt{\tan x}$ (B) $\frac{\sec^2 x}{2\sqrt{\tan x}}$ (C) $2\tan x$ (D) $\frac{\sec x}{2\sqrt{\tan x}}$

74. $\frac{d}{dx}[\log(\cos e^x)] = ?$

(A) $e^x + \tan x$ (B) $-e^x\tan(e^x)$

(C) $e^x\cot x$ (D) $e^x\cos x$

75. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$ then $A^2 =$

(A) a unit matrix (B) A

(C) a null matrix (D) –A

76. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ where $A^2 = B$, then the value of $\alpha$ is :

(A) 1 (B) –1

(C) 4

(D) we can't calculate the value of $\alpha$

77. If A, B and C are three events independent of each other then $P(A\cap B\cap C) =$

(A) $P(A) + P(B) + P(C)$ (B) $P(A) - P(B) + P(C)$

(C) $P(A) + P(B) - P(A\cap B)$ (D) $P(A)\,P(B)\,P(C)$

78. If $P(A) = \frac{3}{8}$ : $P(B) = \frac{1}{2}$ and $P(A\cap B) = \frac{1}{4}$ then $P(A\cup B) =$ .....

(A) 0 (B) $\frac{5}{8}$ (C) 1 (D) 4

79. The direction cosines of two straight lines are $l, m, n$ and $l_1, m_1, n_1$. The lines will be parallel if :

(A) $ll_1 + mm_1 + nn_1 = 0$ (B) $\frac{l}{l_1} = \frac{m}{m_1} = \frac{n}{n_1}$

(C) $\frac{l}{l_1} + \frac{m}{m_1} + \frac{n}{n_1} = 0$ (D) $l_1 + mm_1 + nn_1 = 1$

80. If a line make angle $\alpha, \beta, \gamma$ with the positive coordinate axes then :

(A) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 0$

(B) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$

(C) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

(D) $\sin^2\theta = \sin^2\beta = \sin^2\gamma$

81. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = ?$

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{3}$

82. The value of the determinant $\begin{bmatrix} 7 & 11 & 13 \\ 17 & 19 & 23 \\ 29 & 31 & 37 \end{bmatrix}$ is :

(A) –36 (B) 36
(C) 20 (D) None of these

83. If $\omega \neq 1, \omega^3 = 1$ and $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$ then $x =$

(A) 0 (B) $\omega$
(C) $\omega^2$ (D) None of these

84. The direction cosines of the y-axis are :
(A) (0, 0, 0) (B) (1, 0, 0)
(C) (0, 1, 0) (D) (0, 0, 1)

85. $\hat{n}^2 =$

(A) 0 (B) 1 (C) –1 (D) $\vec{n}$

86. $[\vec{a}\ \vec{a}\ \vec{a}] =$

(A) 1 (B) 0 (C) $\vec{a}$ (D) –1

87. The order of the differential equation $2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is :

(A) 2 (B) 1
(C) 0 (D) not defined

88. If $y = \tan^2 x$, then $\frac{dy}{dx} =$

(A) $\sec^2 x$ (B) $\sec^4 x$
(C) $2 \tan x \sec x$ (D) $2 \tan x \sec^2 x$

89. $\frac{d}{dx}(5^x) =$
(A) $5^x$ (B) $x5^{x-1}$

(C) $\frac{5^x}{\log 5}$ (D) $5^x.\log_e 5$

90. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to =

(A) $\frac{5\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

91. $\tan^{-1} 1 + \tan^{-1}\frac{1}{3} = ?$

(A) $\tan^{-1} 2$ (B) $\tan^{-1} 3$ (C) $\tan^{-1}\frac{4}{3}$ (D) $\tan^{-1}\frac{2}{3}$

92. $\int_{-1}^{1} |x| dx =$

(A) 2 (B) 1 (C) 0 (D) –1

93. $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx =$

(A) log (sin $x$ + cos $x$) (B) $x$
(C) log $x$ (D) log sin (cos $x$)

94. $\vec{k} \times \vec{k} =$

(A) 0 (B) 1 (C) –1 (D) $k^2$

95. $\vec{a} \parallel \vec{b} \Leftrightarrow \vec{a} \times \vec{b}$

(A) 0 (B) +2 (C) 1 (D) –1

96. $\int e^x (\cot x + \log \sin x)\, dx = ?$

(A) $e^x \log \sin x + k$ (B) $e^x \cot x + k$
(C) $e^x \sin x + k$ (D) $e^x \cos x + k$

97. $\int_a^b x^2 dx = ?$

(A) $-\frac{y}{x}$ (B) $\frac{x}{y}$ (C) $-\frac{x}{y}$ (D) $\frac{y}{x}$

98. $\begin{vmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 5 & 6 \end{vmatrix} = ......?$

(A) 5 (B) 7 (C) 0 (D) 9

99. The slope of the normal to the curve $y = 2x^2 + 3 \sin x$ at $x = 0$ is

(A) 3 (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) –3

100. The integrating factor of the different equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is :

(A) $x$ (B) $-x$ (C) $\log x$ (D) $x^2$

## SECTION – B : Non-Objective Type Questions

## SHORT ANSWER TYPE QUESTIONS

**Direction :** *Question Nos. 1 to 30 are of short answer type. Answer only 15 questions from these.* 15 × 2 = 30

1. **Integrate :** $\int \cos^2 x\, dx$.

2. **Integrate :** $\int \frac{x-2}{x^2 - 3x + 2} dx$

3. **Evaluate :** $\int \sqrt{1+\sin 2x}\, dx$

4. **Integrate :** $\int \frac{(\sin x - \cos x)^3}{\sqrt{1-\sin 2x}} dx$

5. **Find** $\int \tan\theta \sec^4\theta\, d\theta$.

6. **Evaluate :** $\int_0^{\pi/2} \cos^2 x\, dx$

7. **Evaluate** $\lim_{x\to\infty}\sum_{r=1}^{n}\frac{r^2}{n^3}$

8. **Find the value of** $\int_2^4 \frac{6x^2-1}{\sqrt{2x^3-x}}dx$

9. **If** $A=\begin{bmatrix}-2\\4\\5\end{bmatrix}$ **and** $B = [\,1\ 3\ -6\,]$. **Find B'A'.**

10. **Find the value of** $x$, **if**

$$\begin{pmatrix}3x+y & -y\\2y-x & 3\end{pmatrix}=\begin{pmatrix}1 & 2\\-5 & 3\end{pmatrix}$$

11. **Evaluate :** $\begin{vmatrix}\cos 15^\circ & \sin 15^\circ\\ \sin 75^\circ & \cos 75^\circ\end{vmatrix}$

12. **Using properties of determinants, prove the following :**

$$\begin{vmatrix}x & x+y & x+2y\\x+2y & x & x+y\\x+y & x+2y & x\end{vmatrix}=9y^2(x+y)$$

13. **Differentiate :** $\sin(x^2+5)$

14. **If** $y=\sqrt{\sin\sqrt{x}}$, **find** $\frac{dy}{dx}$

15. **If** $y = x^2\cos(\log x)$, **find** $\frac{dy}{dx}$

16. **Solve the following differential equation :**

$$(x^2+xy)\,dy=(x^2+y^2)\,dx.$$

17. **Solve the following differential equation**

$$e^{\frac{dy}{dx}}=x,\ y(1)=1$$

18. **If** $\hat{a},\hat{b}$ **and** $\hat{c}$ **are mutually perpendicular unit vectors, then find the value of** $|2\hat{a}+\hat{b}+\hat{c}|$

19. **If** $\bar{a}$ **and** $\bar{b}$ **are two unit vectors such that** $\bar{a}+\bar{b}$ **is also a unit vector, then find the angle between** $\bar{a}$ **and** $\bar{b}$.

20. **Find the equaition of the plane through** $P(1, 4, -2)$ **that is parallel to the plane** $-2x+y-3z=0$.

21. **If** $A$ **and** $B$ **are independent events and** $P(A\cap B)=\frac{1}{8}$, $P(A'\cap B')=\frac{3}{8}$, **find** $P(A)$ **and** $P(B)$

22. **Prove that :** $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$

23. **Prove that :**

$$\sin^{-1}x+\sin^{-1}y=\sin^{-1}\left|x\sqrt{(1-y^2)}+y\sqrt{(1-x^2)}\right|$$

24. **Show that :**

$$\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$$

25. **Find the value of**

$$\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$$

26. **Differentiate :** $\cot^{-1}\sqrt{x}$

27. **If** $\vec{a}=(2,4,-5)$ **and** $\vec{b}=(2,2,2)$, **then find the angle between the vectors** $\vec{a}$ **and** $\vec{b}$.

28. **Find the equation of the plane whose intercepts on the axes of** $x$, $y$ **and** $z$ **are respectively 3, 4 and –5.**

29. **Evaluate the determinant** $\begin{vmatrix}16 & 9 & 7\\23 & 16 & 7\\32 & 19 & 13\end{vmatrix}$

30. **Solve :** $y(1+xy)\,dx - x\,dy = 0$

## LONG ANSWER TYPE QUESTIONS

**Direction :** *Question Nos. 31 to 38 are long answer type question. Answer any 4 questions from these.* 5 × 4 = 20

31. **Find** $\frac{dy}{dx}$, **if** $y^x = x^y$.

32. **Find the area of the region bounded by** $x^2 = 4y$, $y = 2$, $y = 4$ **and the Y-axis in the first quadrant.**

33. **Show that the diagonals of rhombus are at right angles.**

34. **Find the co-ordinates of the point where the line joining the points** $P(1, -2, 3)$ **and** $Q(4, 7, 8)$ **cut the** $xy$**-plane.**

35. **The probabilities of solving a problem for three students are** $\frac{1}{3},\frac{1}{4},\frac{1}{5}$ **respectively. Then find the probability that the problem will be solved.**

36. **Prove that :**

$$\begin{vmatrix}\alpha & \beta & \gamma\\\alpha^2 & \beta^2 & \gamma^2\\\beta+\alpha & \gamma+\alpha & \alpha+\beta\end{vmatrix}=(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$$

37. **Solve the following LPP graphically**

Maximize $z = 8x + 7y$

subject to $3x + y \le 66$

$x + y \le 45$

$x \le 20$

$y \le 40$

$x, y \ge 0$

38. **Prove that :** $\int_0^{\pi/2}\log\sin x\,dx=-\frac{\pi}{2}\log 2$

# ANSWER WITH EXPLANATIONS

## SECTION – A

### OMR ANSWER-SHEET

| | |
|---|---|
| 1. Ⓐ Ⓑ Ⓒ Ⓓ | 51. Ⓐ Ⓑ Ⓒ Ⓓ |
| 2. Ⓐ Ⓑ Ⓒ Ⓓ | 52. Ⓐ Ⓑ Ⓒ Ⓓ |
| 3. Ⓐ Ⓑ Ⓒ Ⓓ | 53. Ⓐ Ⓑ Ⓒ Ⓓ |
| 4. Ⓐ Ⓑ Ⓒ Ⓓ | 54. Ⓐ Ⓑ Ⓒ Ⓓ |
| 5. Ⓐ Ⓑ Ⓒ Ⓓ | 55. Ⓐ Ⓑ Ⓒ Ⓓ |
| 6. Ⓐ Ⓑ Ⓒ Ⓓ | 56. Ⓐ Ⓑ Ⓒ Ⓓ |
| 7. Ⓐ Ⓑ Ⓒ Ⓓ | 57. Ⓐ Ⓑ Ⓒ Ⓓ |
| 8. Ⓐ Ⓑ Ⓒ Ⓓ | 58. Ⓐ Ⓑ Ⓒ Ⓓ |
| 9. Ⓐ Ⓑ Ⓒ Ⓓ | 59. Ⓐ Ⓑ Ⓒ Ⓓ |
| 10. Ⓐ Ⓑ Ⓒ Ⓓ | 60. Ⓐ Ⓑ Ⓒ Ⓓ |
| 11. Ⓐ Ⓑ Ⓒ Ⓓ | 61. Ⓐ Ⓑ Ⓒ Ⓓ |
| 12. Ⓐ Ⓑ Ⓒ Ⓓ | 62. Ⓐ Ⓑ Ⓒ Ⓓ |
| 13. Ⓐ Ⓑ Ⓒ Ⓓ | 63. Ⓐ Ⓑ Ⓒ Ⓓ |
| 14. Ⓐ Ⓑ Ⓒ Ⓓ | 64. Ⓐ Ⓑ Ⓒ Ⓓ |
| 15. Ⓐ Ⓑ Ⓒ Ⓓ | 65. Ⓐ Ⓑ Ⓒ Ⓓ |
| 16. Ⓐ Ⓑ Ⓒ Ⓓ | 66. Ⓐ Ⓑ Ⓒ Ⓓ |
| 17. Ⓐ Ⓑ Ⓒ Ⓓ | 67. Ⓐ Ⓑ Ⓒ Ⓓ |
| 18. Ⓐ Ⓑ Ⓒ Ⓓ | 68. Ⓐ Ⓑ Ⓒ Ⓓ |
| 19. Ⓐ Ⓑ Ⓒ Ⓓ | 69. Ⓐ Ⓑ Ⓒ Ⓓ |
| 20. Ⓐ Ⓑ Ⓒ Ⓓ | 70. Ⓐ Ⓑ Ⓒ Ⓓ |
| 21. Ⓐ Ⓑ Ⓒ Ⓓ | 71. Ⓐ Ⓑ Ⓒ Ⓓ |
| 22. Ⓐ Ⓑ Ⓒ Ⓓ | 72. Ⓐ Ⓑ Ⓒ Ⓓ |
| 23. Ⓐ Ⓑ Ⓒ Ⓓ | 73. Ⓐ Ⓑ Ⓒ Ⓓ |
| 24. Ⓐ Ⓑ Ⓒ Ⓓ | 74. Ⓐ Ⓑ Ⓒ Ⓓ |
| 25. Ⓐ Ⓑ Ⓒ Ⓓ | 75. Ⓐ Ⓑ Ⓒ Ⓓ |
| 26. Ⓐ Ⓑ Ⓒ Ⓓ | 76. Ⓐ Ⓑ Ⓒ Ⓓ |
| 27. Ⓐ Ⓑ Ⓒ Ⓓ | 77. Ⓐ Ⓑ Ⓒ Ⓓ |
| 28. Ⓐ Ⓑ Ⓒ Ⓓ | 78. Ⓐ Ⓑ Ⓒ Ⓓ |
| 29. Ⓐ Ⓑ Ⓒ Ⓓ | 79. Ⓐ Ⓑ Ⓒ Ⓓ |
| 30. Ⓐ Ⓑ Ⓒ Ⓓ | 80. Ⓐ Ⓑ Ⓒ Ⓓ |
| 31. Ⓐ Ⓑ Ⓒ Ⓓ | 81. Ⓐ Ⓑ Ⓒ Ⓓ |
| 32. Ⓐ Ⓑ Ⓒ Ⓓ | 82. Ⓐ Ⓑ Ⓒ Ⓓ |
| 33. Ⓐ Ⓑ Ⓒ Ⓓ | 83. Ⓐ Ⓑ Ⓒ Ⓓ |
| 34. Ⓐ Ⓑ Ⓒ Ⓓ | 84. Ⓐ Ⓑ Ⓒ Ⓓ |
| 35. Ⓐ Ⓑ Ⓒ Ⓓ | 85. Ⓐ Ⓑ Ⓒ Ⓓ |
| 36. Ⓐ Ⓑ Ⓒ Ⓓ | 86. Ⓐ Ⓑ Ⓒ Ⓓ |
| 37. Ⓐ Ⓑ Ⓒ Ⓓ | 87. Ⓐ Ⓑ Ⓒ Ⓓ |
| 38. Ⓐ Ⓑ Ⓒ Ⓓ | 88. Ⓐ Ⓑ Ⓒ Ⓓ |
| 39. Ⓐ Ⓑ Ⓒ Ⓓ | 89. Ⓐ Ⓑ Ⓒ Ⓓ |
| 40. Ⓐ Ⓑ Ⓒ Ⓓ | 90. Ⓐ Ⓑ Ⓒ Ⓓ |
| 41. Ⓐ Ⓑ Ⓒ Ⓓ | 91. Ⓐ Ⓑ Ⓒ Ⓓ |
| 42. Ⓐ Ⓑ Ⓒ Ⓓ | 92. Ⓐ Ⓑ Ⓒ Ⓓ |
| 43. Ⓐ Ⓑ Ⓒ Ⓓ | 93. Ⓐ Ⓑ Ⓒ Ⓓ |
| 44. Ⓐ Ⓑ Ⓒ Ⓓ | 94. Ⓐ Ⓑ Ⓒ Ⓓ |
| 45. Ⓐ Ⓑ Ⓒ Ⓓ | 95. Ⓐ Ⓑ Ⓒ Ⓓ |
| 46. Ⓐ Ⓑ Ⓒ Ⓓ | 96. Ⓐ Ⓑ Ⓒ Ⓓ |
| 47. Ⓐ Ⓑ Ⓒ Ⓓ | 97. Ⓐ Ⓑ Ⓒ Ⓓ |
| 48. Ⓐ Ⓑ Ⓒ Ⓓ | 98. Ⓐ Ⓑ Ⓒ Ⓓ |
| 49. Ⓐ Ⓑ Ⓒ Ⓓ | 99. Ⓐ Ⓑ Ⓒ Ⓓ |
| 50. Ⓐ Ⓑ Ⓒ Ⓓ | 100. Ⓐ Ⓑ Ⓒ Ⓓ |

### ANSWER

| | | | | |
|---|---|---|---|---|
| 1. (D) | 2. (D) | 3. (C) | 4. (B) | 5. (B) |
| 6. (C) | 7. (B) | 8. (C) | 9. (C) | 10. (B) |
| 11. (B) | 12. (B) | 13. (C) | 14. (B) | 15. (A) |
| 16. (A) | 17. (D) | 18. (A) | 19. (A) | 20. (A) |
| 21. (A) | 22. (C) | 23. (B) | 24. (B) | 25. (B) |
| 26. (D) | 27. (C) | 28. (B) | 29. (A) | 30. (C) |
| 31. (B) | 32. (B) | 33. (B) | 34. (A) | 35. (D) |
| 36. (C) | 37. (B) | 38. (D) | 39. (A) | 40. (B) |
| 41. (D) | 42. (D) | 43. (B) | 44. (C) | 45. (C) |
| 46. (C) | 47. (D) | 48. (B) | 49. (B) | 50. (C) |
| 51. (B) | 52. (D) | 53. (B) | 54. (B) | 55. (C) |
| 56. (B) | 57. (B) | 58. (B) | 59. (A) | 60. (C) |
| 61. (C) | 62. (A) | 63. (B) | 64. (C) | 65. (C) |
| 66. (B) | 67. (B) | 68. (C) | 69. (C) | 70. (B) |
| 71. (B) | 72. (B) | 73. (B) | 74. (B) | 75. (A) |
| 76. (D) | 77. (D) | 78. (B) | 79. (B) | 80. (C) |
| 81. (A) | 82. (B) | 83. (A) | 84. (C) | 85. (B) |
| 86. (B) | 87. (A) | 88. (D) | 89. (D) | 90. (A) |
| 91. (A) | 92. (B) | 93. (B) | 94. (A) | 95. (A) |
| 96. (A) | 97. (A) | 98. (C) | 99. (B) | 100. (C) |

## SECTION – B

**1.** Let, I $= \int \cos^2 x\, dx = \frac{1}{2}\int 2\cos^2 x\, dx$

$= \frac{1}{2}\int (1+\cos 2x) = \frac{1}{2}\int dx + \int \cos 2x\, dx$

$= \frac{1}{2}\times x + \frac{1}{4}\sin 2x = \frac{x}{2} + \frac{1}{4}\sin 2x + c$; **Ans.**

**2.** Let, I $= \int \frac{x-2}{x^2-3x+2}dx = \int \frac{x-2}{(x-2)(x-1)}dx$

$= \int \frac{dx}{x-1} = \log(x-1) + C$; **Ans.**

**3.** Let, I $= \int \sqrt{1+\sin 2x}\, dx$

$= \int \sqrt{\cos^2 x + \sin^2 x + 2\sin x \cdot \cos x}\, dx$

$= \int \sqrt{(\cos x + \sin x)^2}\, dx$

$= \int (\cos x + \sin x) dx$

$= \int \cos x dx + \int \sin x dx$

$= \sin x - \cos x + C$ **Ans.**

**4.** Let, I $= \int \frac{(\sin x - \cos x)^3}{\sqrt{1-\sin 2x}}dx$

$= \int \frac{(\sin x - \cos x)^3}{\sqrt{\sin^2 x + \cos^2 x - 2\sin x.\cos x}}dx$

[$\because$ $\sin 2x = 2\sin x \,.\, \cos x$]

$= \int \frac{(\sin x - \cos x)^3}{\sqrt{(\sin x - \cos)^2}}dx = \int \frac{(\sin x - \cos x)^3}{(\sin x - \cos x)}dx$

$= \int (\sin x - \cos x)^2\, dx$
$= \int (\sin^2 x + \cos^2 x - 2\sin x.\cos x)dx$
$= \int (1 - \sin 2x)\, dx = \int dx - \int \sin 2x\, dx$

$= x - \frac{-\cos 2x}{2} + c = x + \frac{\cos 2x}{2} + c$; **Ans.**

5. Let $z = \tan\theta$, then $dz = \sec^2\theta\, d\theta$

Now, $\int \tan\theta \sec^4\theta\, d\theta = \int \tan\theta \sec^2\theta \cdot \sec^2\theta\, d\theta$
$= \int \tan\theta\,(1 + \tan^2\theta)\sec^2\theta\, d\theta$
$= \int z(1 + z^2)\, dz = \int (z + z^3)\, dz$

$= \frac{z^2}{2} + \frac{z^4}{4} + c$

$= \frac{\tan^2\theta}{2} + \frac{\tan^4\theta}{4} + c.$

6. Let, $I = \int_0^{\pi/2} \cos^2 x\, dx$

$= \frac{1}{2}\int_0^{\pi/2} (\cos 2x + 1)dx$

$= \frac{1}{2}\int_0^{\pi/2} \cos 2x\, dx + \frac{1}{2}\int_0^{\pi/2} dx$

$= \frac{1}{2}\left[\frac{\sin 2x}{2}\right]_0^{\pi/2} + \frac{1}{2}[x]_0^{\pi/2}$

$= \frac{1}{4}(0-0) + \frac{1}{2}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{4}$ **Ans.**

7. $\lim_{x\to\infty} \sum_{r=1}^{n} \frac{r^2}{n^3}$

$= \lim_{x\to\infty}\left(\frac{1}{n^3} + \frac{2}{n^3} + \ldots\ldots + \frac{n^2}{n^3}\right)$

$= \lim_{x\to\infty}\left(\frac{1}{n^3} + \frac{2}{n^3} + \ldots\ldots + \frac{1}{n}\right)$

$= 0$; **Ans.**

8. Let, $I = \int_2^4 \frac{6x^2 - 1}{\sqrt{2x^3 - x}}dx$

Putting $z^2 = 2x^3 - x$

D.w.r. to $x$, $2z\frac{dz}{dx} = 6x^2 - 1$

Or, $2zdz = (6x^2 - 1)\, dx$

$\therefore\ I = \int_2^4 \frac{2zdz}{z} = 2\int_2^4 dz$

$= 2[z]_2^4 = [2\sqrt{2x^3 - x}]_2^4$

$= 2\left[\sqrt{2\times 4^3 - 4} - \sqrt{2\times 2^3 - 2}\right]$

$= 2\left[\sqrt{2\times 64 - 4} - \sqrt{2\times 8 - 2}\right]$

$= 2\left[\sqrt{128 - 4} - \sqrt{16 - 2}\right]$

$= 2\left[\sqrt{124} - \sqrt{14}\right]$; **Ans.**

9. $\because\ A' = [-2\ 4\ 5]$ and $B' = \begin{bmatrix}1\\3\\-6\end{bmatrix}$

$\therefore\ B'A' = \begin{bmatrix}1\\3\\-6\end{bmatrix}[-2\ 4\ 5]$

$= \begin{bmatrix}-2 & 4 & 5\\-6 & 12 & 15\\12 & -24 & -30\end{bmatrix}$

10. We have

$\begin{pmatrix}3x+y & -y\\2y-x & 3\end{pmatrix} = \begin{pmatrix}1 & 2\\-5 & 3\end{pmatrix}$

By definition of equality of two matrices, we have

$3x + y = 1$ ... (1)

$2y - x = -5$ ... (2)

and $-y = 2 \Rightarrow y = -2$

Putting $y$ in (1), we get

$3x - 2 = 1 \Rightarrow 3x = 3 \Rightarrow x = 1$

$\therefore\ x = 1, y = -2$

11. We have $\begin{vmatrix}\cos 15^\circ & \sin 15^\circ\\ \sin 75^\circ & \cos 75^\circ\end{vmatrix}$

$= \cos 75^\circ . \cos 15^\circ - \sin 75^\circ \sin 15^\circ$
$= \cos(75^\circ + 15^\circ)$
$= \cos 90^\circ$
$= 0$

12. LHS $= \begin{vmatrix}x & x+y & x+2y\\x+2y & x & x+y\\x+y & x+2y & x\end{vmatrix}$

Operate $= C_1 \to C_1 + C_2 + C_3$

$= \begin{vmatrix}3x+3y & x+y & x+2y\\3x+3y & x & x+y\\3x+3y & x+2y & x\end{vmatrix}$

$= 3(x+y)\begin{vmatrix}1 & x+y & x+2y\\1 & x & x+y\\1 & x+2y & x\end{vmatrix}$

Operate : $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_2$

$= 3(x+y)\begin{vmatrix}1 & x+y & x+2y\\0 & -y & -y\\0 & 2y & -y\end{vmatrix}$

$= 3(x+y).y.y\begin{vmatrix}1 & x+y & x+2y\\0 & -1 & -1\\0 & 2 & -1\end{vmatrix}$

Expand by $C_1$

$= 3y^2(x+y).1(1+2) = 9y^2(x+y)$
= RHS

13. Let $y = \sin(x^2 + 5)$

Then $\frac{dy}{dx} = \frac{d\sin(x^2+5)}{dx}$

$= \frac{d\sin(x^2+5)}{d(x^2+5)}\cdot\frac{d(x^2+5)}{dx}$

$= \cos(x^2+5)\cdot\left[\frac{d(x^2)}{dx} + \frac{d(5)}{dx}\right]$

$= \cos(x^2+5)\cdot(2x+0) = 2x\cos(x^2+5)$ **Ans.**

14. $\frac{dy}{dx} = \frac{d}{dx}(\sqrt{\sin\sqrt{x}})$

$= \frac{d\sqrt{\sin\sqrt{x}}}{d\sin\sqrt{x}} \cdot \frac{d\sin\sqrt{x}}{d\sqrt{x}} \cdot \frac{d\sqrt{x}}{dx}$

$= \frac{1}{2\sqrt{\sin\sqrt{x}}} \cdot \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

$= \frac{\cos\sqrt{x}}{4\sqrt{x\sin\sqrt{x}}}$ **Ans.**

15. $\frac{dy}{dx} = \frac{d}{dx}[x^2\cos(\log x)]$

$= x^2\frac{d}{dx}\{\cos(\log x)\} + \cos(\log x)\frac{d(x^2)}{dx}$

$= x^2 . \frac{d\cos(\log x)}{d\log x} \cdot \frac{d}{dx}(\log x) + \cos(\log x) \cdot 2x$

$= x^2\{-\sin(\log x)\} \cdot \frac{1}{x} + 2x\cos(\log x)$

$= -x\sin(\log x) + 2x\cos(\log x)$ **Ans.**

16. The given D.E. is

$(x^2 + xy)\,dy + (x^2 + y^2)\,dx$

$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$ ... (1)

This is a homogeneous linear D.E.

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ Equation (1) $\Rightarrow$

$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{x^2 + x.vx} = \frac{1+v^2}{1+v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{1+v} - v = \frac{1-v}{1+v}$

$\Rightarrow \frac{1+v}{1-v}dv = \frac{dx}{x}$

Integrating, we get

$\int\left[-1 + \frac{2}{1-v}\right]dv = \int\frac{dx}{x} + c$

$\Rightarrow -v - 2\log(1-v) = \log x + c$

$\Rightarrow -\frac{y}{x} - 2\log\left(1 - \frac{y}{x}\right) = \log x + c$

Thus is the sol. of (1).

17. Given D.E. is

$e^{\frac{dy}{dx}} = x$

Taking log both sides, we get

$\log e^{\frac{dy}{dx}} = \log x$

$\frac{dy}{dx}\log e = \log x$

$\therefore \int dy = \int \log x\,dx$

$y = \log x . x - \int\frac{1}{x}x\,dx + cv$

$y = x\log x - \int dx + c$

$y = x\log x - x + c$

Given $x = 1, y = 1$

$1 = 1(\log 1) - 1 + c$

$1 = 0 - 1 + c \Rightarrow c = 2$

$\therefore$ Required solution is

$y = x\log x - x + 2$

18. $\because \hat{a}, \hat{b}$ and $\hat{c}$ are perpendicular unit vector.

Hence, $\hat{a}\cdot\hat{b} = \hat{b}\cdot\hat{c} = \hat{c}\cdot\hat{a} = 0$

and $|\hat{a}| = |\hat{b}| = |\hat{c}| = 1$

So, $|2\hat{a} + \hat{b} + \hat{c}|^2 = (2\hat{a} + \hat{b} + \hat{c})(2\hat{a} + \hat{b} + \hat{c})$

$[\because \hat{a}\cdot\hat{a} = |\hat{a}|^2]$

$= 4(\hat{a}\cdot\hat{a}) + 2(\hat{a}\cdot\hat{b}) + 2(\hat{a}\cdot\hat{c}) + 2(\hat{b}\cdot\hat{a})$
$+ (\hat{b}\cdot\hat{b}) + (\hat{b}\cdot\hat{c}) + 2(\hat{c}\cdot\hat{a}) + (\hat{c}\cdot\hat{b}) + (\hat{c}\cdot\hat{c})$

$= 4(|\hat{a}|^2) + 2(0) + 2(0) + 2(0) + (|\hat{b}|^2) + (0) + 2(0) + (0) + (|\hat{c}|^2)$

$= 4 \times 1 + 1 + 1 = 6$

$\therefore |2\hat{a} + \hat{b} + \hat{c}| = \sqrt{6}$ **Ans.**

19. $\because \bar{a} + \bar{b}$ is be unit vector.

$\Rightarrow |\bar{a} + \bar{b}| = 1$

$\Rightarrow |\bar{a} + \bar{b}|^2 = 1^2 = 1$

$\Rightarrow a^2 + b^2 + 2\bar{a}.\bar{b} = 1$

$\Rightarrow 1^2 + 1^2 + 2ab\cos\theta = 1$

$\Rightarrow 1 + 1 + 2.1.1\cos\theta = 1$

$\Rightarrow 2\cos\theta = -1$

$\Rightarrow \cos\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\therefore \theta = \frac{2\pi}{3}$ **Ans.**

20. The equation of any plane parallel to the given plane $-2x + y - 3x = 0$ is

$-2x + y - 3z + k = 0$ ...(1)

If plane (1) passes throught $P(1, 4, -2)$, then

$(-2)(1) + 4 - 3(-2) + k = 0$

$\Rightarrow -2 + 4 + 6 + k = 0 \quad \therefore k = -8$

substituting $k = -8$ in (1), we get

$-2x + y - 3z - 8 = 0$

21. Let $P(A) = x$ and $P(B) = y$

$\because A$ and $B$ are independent events

$\therefore P(A \cap B) = P(A) \cdot P(B)$

$\Rightarrow xy = \frac{1}{8}$ ... (1)

Given, $P(A' \cap B') = \frac{3}{8}$

$\therefore \frac{3}{8} = P(A')\cdot P(B')$

$\Rightarrow \frac{3}{8} = (1-x)(1-y) = 1 - x - y + xy$

$\Rightarrow x + y - xy = \frac{5}{8}$ ...(2)

Solving (1) and (2) we get,

$$x = \frac{1}{2} \text{ and } y = \frac{1}{4}$$

$\therefore \quad P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{4}$ **Ans.**

**22.** Let $\tan^{-1} x = \alpha$ and $\tan^{-1} y = \beta$

$\therefore \quad x = \tan \alpha$ and $y = \tan \beta$

Now $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta} = \frac{x+y}{1-xy}$

$$\Rightarrow \quad \alpha + \beta = \tan^{-1}\frac{x+y}{1-xy}$$

$\therefore \quad \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$. **Proved**

**23.** $\because \quad \sin(A+B) = \sin A \cos B + \cos A \sin B$

$$= \sin A\sqrt{1-\sin^2 B} + \sin B\sqrt{1-\sin^2 A}$$

Let, $\sin A = x \Rightarrow A = \sin^{-1} x$ and $\sin B = y \Rightarrow B = \sin^{-1} y$

$\therefore \quad \sin(A+B) = x\sqrt{1-y^2} + y\sqrt{1-x^2}$

$$\Rightarrow \quad A + B = \sin^{-1}\left|x\sqrt{1-y^2} + y\sqrt{1-x^2}\right|$$

$\therefore \quad \sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{(1-y^2)} + y\sqrt{(1-x^2)}\right]$.

**Proved**

**24.** $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right)$

Let $\cos^{-1}\frac{a}{b} = 2\theta \quad \Rightarrow \quad \frac{a}{b} = \cos 2\theta$

$$\Rightarrow \quad \tan\left(\frac{\pi}{4} + \frac{1}{2}\times 2\theta\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\times 2\theta\right)$$

$$= \tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right)$$

$$= \frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}$$

$$= \frac{(1+\tan\theta)^2 + (1-\tan\theta)^2}{(1-\tan\theta)(1+\tan\theta)}$$

$$= \frac{1+\tan^2\theta + 2\tan\theta + 1 + \tan^2\theta - 2\tan\theta}{1-\tan^2\theta}$$

$$= 2\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$$

$$= \frac{2}{\frac{1-\tan^2\theta}{1+\tan^2\theta}} = \frac{2}{\cos 2\theta} = \frac{2b}{a}$$ **Ans.**

**25.** $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$$= \tan^{-1}\sqrt{3} - (\pi - \sec^{-1} 2)$$

$$= \frac{\pi}{3} - \pi + \frac{\pi}{3} \qquad \left(\because \cos\frac{\pi}{3} = \frac{1}{2} \,\&\, \tan\frac{\pi}{3} = \sqrt{3}\right)$$

$= -\frac{\pi}{3}$ **Ans.**

**26.** Let $y = \cot^{-1}\sqrt{x}$

$$\frac{dy}{dx} = \frac{d(\cot^{-1}\sqrt{x})}{dx} = \frac{d\cot^{-1}\sqrt{x}}{d\sqrt{x}} \cdot \frac{d\sqrt{x}}{dx}$$

$$= -\frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}}$$

$= -\frac{1}{2\sqrt{x}(1+x)}$ **Ans.**

**27.** Here $\vec{a} = (2, 3, -5) = (2\vec{i} + 3\vec{j} - 5\vec{k})$

and $\vec{b} = (2, 2, 2) = (2\vec{i} + 2\vec{j} + 2\vec{k})$

$\therefore \quad |\vec{a}| = \sqrt{2^2 + 3^2 + (-5)^2} = \sqrt{38}$

$|\vec{b}| = \sqrt{2^2 + 2^2 + 2^2} = \sqrt{12}$

and $\vec{a}\cdot\vec{b} = (2\vec{i} + 3\vec{j} - 5\vec{k})\cdot(2\vec{i} + 2\vec{j} + 2\vec{k})$

$= 2 \times 2 + 3 \times 2 + (-5) \times 2$

$= 4 + 6 - 10$

$= 10 - 10 = 0$

Hence, $\vec{a}$ and $\vec{b}$ are perpendicular and angle between them is 90°. **Ans.**

**28.** We know that equation of the plane in intercepts

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad \text{...(i)}$$

Here, $a = 3$, $b = 4$, $c = -5$

Required equation of the plane,

$$\frac{x}{3} + \frac{y}{4} + \frac{z}{-5} = 1$$

$$\Rightarrow \quad \frac{20x + 15y + (-12z)}{60} = 1$$

$\Rightarrow \quad 20x + 15y - 12z = 1 \times 12$

$\therefore \quad 20x + 15y - 12z = 12$ **Ans.**

**29.** Determinant $= \begin{vmatrix} 16 & 9 & 7 \\ 23 & 16 & 7 \\ 32 & 19 & 13 \end{vmatrix}$

$$= 16\begin{vmatrix} 16 & 7 \\ 19 & 13 \end{vmatrix} - 9\begin{vmatrix} 23 & 7 \\ 32 & 13 \end{vmatrix} + 7\begin{vmatrix} 23 & 16 \\ 32 & 19 \end{vmatrix}$$

$= 16(16 \times 13 - 7 \times 19) - 9(23 \times 13 - 7 \times 32) + 7(23 \times 19 - 16 \times 32)$

$= 16(208 - 133) - 9(299 - 224) + 7(437 - 512)$

$= 16 \times 75 - 9 \times 75 + 7 \times (-75)$

$= 1200 - 675 - 525$

$= 1200 - 1200 = 0$ **Ans.**

**30.** Here $y\,dx - x\,dy + xy^2\,dx = 0$

$$\Rightarrow \frac{y\,dx - x\,dy}{y^2} + x\,dx = 0$$

$$\Rightarrow d\left(\frac{x}{y}\right) + x\,dx = 0$$

Interating, we have $\frac{x}{y} + \frac{x^2}{2} = k$ (where $k$ = constant);

**Ans.**

**31.** Given, $y^x = x^y$

Taking log both sides, we get

$\log y^x = \log x^y \Rightarrow x \log y = y \log x$

Differentiating both sides w.r.t. $x$,

$$\frac{d}{dx}(x \log y) = \frac{d}{dx}(y \log x)$$

$$\Rightarrow x\left(\frac{1}{y}\right)\frac{dy}{dx} + (\log y) = y\frac{1}{x} + (\log x)\frac{dy}{dx}$$

$$\Rightarrow \frac{x}{y}\frac{dy}{dx} - (\log x)\frac{dy}{dx} = \frac{y}{x} - \log y$$

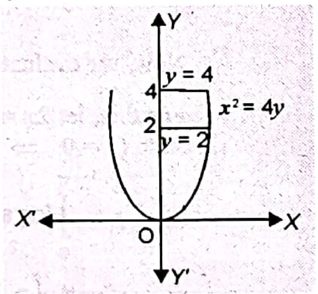
$$\Rightarrow \left(\frac{x}{y} - \log x\right)\frac{dy}{dx} = \frac{y}{x} - \log y$$

$$\Rightarrow \left(\frac{x - y\log x}{y}\right)\frac{dy}{dx} = \frac{y - x\log y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y - x\log y}{x - y\log x}\right)$$

**32.** The given curve $x^2 = 4y$ is a parabola which is symmetrical about Y-axis ($\because$ it contains even power of $x$) only and passes through the origin.

The area of the region bounded by the curve $x^2 = 4y, y = 2$ and $y = 4$ and the Y-axis is shown in the figure. Required area (shaded region)




$$= \int_{y=a}^{y=b} |x|\,dy \quad \text{(Here, } |x| = \sqrt{4y} \text{ and } a = 2, b = 4\text{)}$$

$$= \int_2^4 |x|\,dy \quad \text{(Considering the elementary strip on Y-axis)}$$

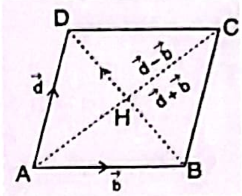
$$= \int_2^4 2\sqrt{y}\,dy \quad (\because x^2 = 4y \quad \therefore |x| = 2\sqrt{y})$$

$$= 2\left[\frac{y^{3/2}}{3/2}\right]_2^4 = \frac{4}{3}[4^{3/2} - 2^{3/2}]$$

$$= \frac{4}{3}[8 - 2\sqrt{2}] = \frac{8}{3}[4 - \sqrt{2}] \text{ sq unit}$$

**33.** Let $ABCD$ be a rhombus..

Take $A$ as origin Let $\vec{b}$ and $\vec{d}$ be the position vectors of $B$ and $D$ respectively. Then the position vector of $C$ *i.e.*,




$\vec{AC} = \vec{b} + \vec{d}$

Now, $AB = AD$

or $AB^2 = AD^2$ or $b^2 = d^2$

Also $\vec{BD} = \vec{AD} - \vec{AB} = \vec{d} - \vec{b}$

Now, $\vec{AC} \cdot \vec{BD} = (\vec{b} + \vec{d}) \cdot (\vec{d} - \vec{b}) = \vec{b} \cdot \vec{d} + d^2 - b^2 - \vec{d} \cdot \vec{b}$

$= d^2 - b^2 = 0 \quad [\because \vec{b} \cdot \vec{d} = \vec{d} \cdot \vec{b}$ and $b^2 = d^2]$

Hence $AC \perp BD$ **Proved**

**34.** Equation of $xy$ plane is $z = 0$

The equation of line is $\frac{x-1}{3} = \frac{y+2}{9} = \frac{z-3}{5}$

any point on this line is $(1 + 3\lambda, -2 + 9\lambda, 3 + 5\lambda)$ this is lies on the $xy$ plane

$\Rightarrow 3 + 5\lambda = 0 \Rightarrow 5\lambda = -3 \therefore \lambda = -\frac{3}{5}$

Required point is $\left(1 + 3\left(-\frac{3}{5}\right), -2 + 9\left(-\frac{3}{5}\right), 0\right)$

$$= \left(1 - \frac{9}{5}, -2 - \frac{27}{5}, 0\right) = \left(-\frac{4}{5}, -\frac{37}{5}, 0\right)$$

**35.** Let the three students be named A, B and C respectively. Let $E_1, E_2, E_3$ be the events that the problem is solved by A, B, C respectively then,

$P(E_1) = \frac{1}{3}$

$P(E_2) = \frac{1}{4}$

$P(E_3) = \frac{1}{5}$

$P(\bar{E}_1) = 1 - \frac{1}{3} = \frac{2}{3}$, $P(\bar{E}_2) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(\bar{E}_3) = 1 - \frac{1}{5} = \frac{4}{5}$

$\therefore$ P (not solves the problem)

$= P(E_1 \cap E_2 \cap E_3)$

$= P(\bar{E}_1 \times \bar{E}_2 \times \bar{E}_3)$

$= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$

$\therefore$ P (that the problem is solved)

$= 1 - \frac{2}{5} = \frac{5-2}{5} = \frac{3}{5}$

**36.** Let $\Delta = \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\alpha & \gamma+\alpha & \alpha+\beta \end{vmatrix}$

$$= \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \alpha+\beta+\gamma & \alpha+\beta+\gamma & \alpha+\beta+\gamma \end{vmatrix} \quad [R_3 \to R_3 + R_1]$$

$$= (\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$$ [Taking out $(\alpha+\beta+\gamma)$ common from $R_3$]

$$= (\alpha+\beta+\gamma)\begin{vmatrix} \alpha-\beta & \beta-\gamma & \gamma \\ \alpha^2-\beta^2 & \beta^2-\gamma^2 & \gamma^2 \\ 0 & 0 & 1 \end{vmatrix}$$ $\begin{bmatrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{bmatrix}$

$$= (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)\begin{vmatrix} 1 & 1 & \gamma \\ \alpha+\beta & \beta+\gamma & \gamma^2 \\ 0 & 0 & 1 \end{vmatrix}$$

[Taking out $(\alpha-\beta)$ common from $C_1$ and $(\beta-\gamma)$ common from $C_2$]

$$= (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)\cdot 1\begin{vmatrix} 1 & 1 \\ \alpha+\beta & \beta+\gamma \end{vmatrix}$$ [Expanding along $R_3$]

$= (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\beta+\gamma-\alpha-\beta)$

$= (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$ **Proved**

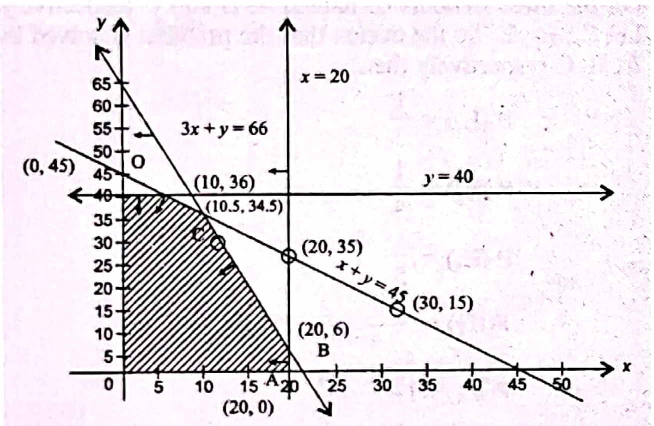
**37.** $3x + y \leq 66$

consider the equation

$3x + y = 66$

$y = 66 - 3x$

| | | |
|---|---|---|
| $x$ | 12 | 10 |
| $y$ | 30 | 36 |




Putting $x = 0$ and $y = 0$ we get

$3 \times 0 + 0 = 0 \leq 66$

so (0, 0) lies in the region $x + 3y \leq 66$

$x + y \leq 45$

consider the equation

$x + y = 45$

| | | |
|---|---|---|
| $x$ | 20 | 30 |
| $y$ | 25 | 15 |

putting $x = 0$ and $y = 0$ we get

$0 + 0 = 0 \leq 45$

so (0, 0) lies in the region $x + 3y \leq 45$

values of $z = 8x + 7y$

(i) At A (20, 0) = 8 × 20 + 7 × 0 = 160

(ii) At B (20, 6) = 8 × 20 + 7 × 6 = 202

(iii) At C (10.5, 34.5) = 8 × 10.5 + 7 × 34.5 = 325.5

(iv) At D (0, 45) = 8 × 0 + 7 × 45 = 315

so maximum value of $z$ is 325.5 at point C (10.5, 34.5) **Ans.**

**38.** Let $I = \int_0^{\pi/2} \log\sin x\, dx$ ... (1)

then $I = \int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right) dx$

Since $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx = \int_0^{\pi/2} \log\cos x dx$ ... (2)

Addiging (1) & (2)

$\Rightarrow \quad 2I = \int_0^{\pi/2} (\log\sin x + \log\cos x)\, dx$

$= \int_0^{\pi/2} \log(\sin x \cos x)\, dx$

$= \int_0^{\pi/2} \log\frac{\sin 2x}{2} dx$

$= \int_0^{\pi/2} \{\log\sin 2x - \log 2\} dx$

$= \int_0^{\pi/2} \log\sin 2x dx - (\log 2)\int_0^{\pi/2} dx$

$= I_1 - \frac{\pi}{2}\log 2$ ... (3)

Now, we evaluate $I_1 = \int_0^{\pi/2} \log\sin 2x\, dx$

For this, let $2x = t$ so that $2dx = dt$.

Also ($x = 0 \Rightarrow t = 0$) and ($x = \pi/2 \Rightarrow t = \pi$)

$\Rightarrow \quad I_1 = \int_0^{\pi} \log\sin t \cdot \frac{dt}{2} = \frac{1}{2}\int_0^{\pi} \log\sin t dt$

Here $\log\sin\left(2.\frac{\pi}{2} - t\right) = \log\sin t$

$\therefore \quad I_1 = \frac{1}{2}.2\int_0^{\pi/2} \log\sin t dt;$

If $(2a - x) = f(x)$,

then $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$

Hence $I_1 = \int_0^{\pi/2} \log\sin t dt = \int_0^{\pi/2} \log\sin x dx = 1$

$\therefore$ From (1), $2I = I - \frac{\pi}{2}\log 2 \Rightarrow I = -\frac{\pi}{2}\log 2$

Hence the result.

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