MODEL PAPER – 5

Time : 3 Hours + 15 Minutes] **[Total Marks : 100**

Instructions to the Candidates :

1. Candidates are required to give their answers in their own words as far as practicable.
2. Figures in the right hand margin indicate full marks.
3. 15 Minutes of extra time has been allotted for the candidates to read the questions carefully.
4. This question paper is divided into two sections—**SECTION – A** and **SECTION – B**.
5. In **SECTION – A** there are **100 Objective Type Question,** out of which only 50 objective question be answered. Darken the circle with blue/black ball pen against the correct option on OMR Sheet provided to you. Do not use **Whitener/Liquid/ Blade/Nail** on OMR paper; otherwise the result will be invalid.
6. In **SECTION – B**, there are **30 Short Answer Type Questions** (each carrying 2 marks), out of which any 15 questions are be answered.
   Apart from this, there are **8 Long Answer Type Question** (Each Carrying 5 marks), out of which 4 Questions are to be answered.
7. Use of any electronic device is prohibited.

SECTION – A : Objective Type Questions

Direction : *There are 100 Objective Type Questions, out of which only 50 objective questions to be answered. Mark the correct option on the OMR Answer Sheet.* 50 × 1 = 50

1. $|3\vec{i}-4\vec{j}-5\vec{k}|=$
   (A) $5\sqrt{2}$ (B) 12
   (C) 2 (D) 9

2. $(3\vec{i}-4\vec{k})^2=$
   (A) 1 (B) 25
   (C) 7 (D) 49

3. $\frac{d}{dx}\left(\sin\frac{4x}{5}\right)=$
   (A) $\frac{4}{5}\cos\frac{4x}{5}$ (B) $-\frac{4}{5}\cos\frac{4x}{5}$
   (C) $\frac{5}{4}\cos\frac{4x}{5}$ (D) $-\frac{5}{4}\cos\frac{4x}{5}$

4. $\frac{d}{dx}\left(2\cos\frac{3x}{4}\right)=$
   (A) $-2\sin\frac{3x}{4}$ (B) $-\frac{3}{8}\sin\frac{3x}{4}$
   (C) $\frac{-3}{4}\sin\frac{3x}{4}$ (D) $\frac{-3}{2}\sin\frac{3x}{4}$

5. $\sin\left(\sin^{-1}\frac{1}{2}\right)=$
   (A) 1 (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) 0

6. The minimum value of $Z=7x+8y$ subject to constraints $3x+4y\le 24,\ x\ge 0, y\ge 0$ is :
   (A) 56 (B) 48
   (C) 0 (D) −12

7. If the direction ratios of two mutually perpendicular lines are 2, 3, 5 and x, y, 4 then $2x+3y=$
   (A) 20 (B) −20
   (C) 30 (D) −30

8. The solution of the differential equation $e^{3x}dx+e^{4y}dy=0$ is :
   (A) $e^{3x+4y}=k$ (B) $e^{3x}+e^{4y}=k$
   (C) $\frac{1}{3}e^{3x}+\frac{1}{4}e^{4y}=k$ (D) $e^{3x}+e^{4y}+e^{3x+4y}=k$

9. $\int\frac{dx}{x^2+5}=$
   (A) $\tan^{-1}\frac{x}{5}+k$ (B) $\tan^{-1}\frac{x}{\sqrt{5}}+k$
   (C) $\frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}}+k$ (D) $\sqrt{5}\tan^{-1}\frac{x}{\sqrt{5}}+k$

10. $\int_{-1}^{1}\log\left(\frac{3+x}{3-x}\right)dx=$
    (A) 0 (B) 1 (C) 2 log 3 (D) 3 log 2

11. $[6\quad 5]\begin{bmatrix}1\\-1\end{bmatrix}=$
    (A) [6 −5] (B) [−5 6] (C) [1] (D) [11]

12. $\int_{-1}^{1}x^{17}\cos^4 x\,dx=$
    (A) 0 (B) 1 (C) $\frac{3}{17}$ (D) $\frac{14}{3}$

13. Which of the following has inverse function ?
    (A) one-one into (B) one-one onto
    (C) many one into (D) many one onto

14. If A = {1, 2, 3}, then the number of all one-one function from A to A is :
    (A) 2 (B) 3 (C) 4 (D) 6

15. $\tan^{-1}1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{2}$ (D) 6π

16. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is :

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$

17. $\frac{d}{dx}(\sin x) =$

(A) $\cos x$ (B) $-\sin x$
(C) $-\cos x$ (D) $\tan x$

18. $\frac{d}{dx}(\tan ax)$

(A) $a \tan ax$ (B) $a \sec^2 ax$
(C) $a \sec x$ (D) $a \cot ax$

19. $\int_2^4 \frac{dx}{x} =$

(A) lgo 2 (B) log 4
(C) – log 2 (D) – log 4

20. $\int \frac{dx}{x+\sqrt{x}} =$

(A) $\log x + \log(1+\sqrt{x}) + C$ (B) $2\log(1+\sqrt{x}) + C$
(C) $\log(1+\sqrt{x}) + C$ (D) $\log\sqrt{x} + C$

21. The maximum value of $Z = 3x + 2y$
Subject to constraints $x + 2y \le 10$
$3x + y \le 15$
$x \ge 0,\ y \ge 0$ is

(A) 0 (B) 15 (C) 10 (D) 18

22. If $x^2y^2 = (x+y)^5$ then $\frac{dy}{dx} =$

(A) $\frac{x}{y}$ (B) $\frac{y}{x}$ (C) $\frac{-y}{x}$ (D) $\frac{-x}{y}$

23. The direction cosines of the x-axis are :

(A) (0, 0, 0) (B) (1, 0, 0)
(C) (0, 1, 0) (D) (0, 0, 1)

24. If l, m, n are the direction consines of a straight line then :

(A) $l^2 + m^2 - n^2 = 1$ (B) $l^2 - m^2 + n^2 = 1$
(C) $l^2 - m^2 - n^2 = 1$ (D) $l^2 + m^2 + n^2 = 1$

25. $\int_0^{\pi/2} \frac{\cos\theta}{\cos\theta + \sin\theta} d\theta =$

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

26. $\int_0^1 e^x dx =$

(A) e (B) $e + 1$ (C) $e - 1$ (D) $1 - e$

27. $\vec{a} \times \vec{b} =$

(A) $\vec{b} \times \vec{a}$ (B) $-\vec{b} \times \vec{a}$ (C) $\vec{a} \cdot \vec{b}$ (D) $\vec{b} \cdot \vec{a}$

28. $|3\vec{i} + 2\vec{j} + 5\vec{k}| =$

(A) 38 (B) $\sqrt{38}$ (C) 10 (D) 30

29. The solution of the differential equation $rdx + ydy = 0$ is :

(A) $x^2 + y^2 = c$ (B) $x^2 - y^2 = c$
(C) $x + y = c$ (D) None of these

30. $\frac{d^2}{dx^2}(\sin 2x) =$

(A) $4 \sin 2x$ (B) $4 \cos^2 2x$
(C) $-4 \sin 2x$ (D) $2 \sin 4x$

31. $\frac{d}{dx}\left\{\lim_{x\to a} \frac{x^n - a^n}{x - a}\right\} =$

(A) $\frac{\pi}{2}$ (B) 0 (C) 1 (D) n

32. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, then :

(A) $(x = -2, y = 8)$ (B) $(x = -2, y = -8)$
(C) $(x = 3, y = -6)$ (D) $(x = -3, y = 6)$

33. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ then $|2A| =$:

(A) $2\,|A|$ (B) $4\,|A|$
(C) $8\,|A|$ (D) None

34. If $P(A \cap B) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A) = \frac{1}{4}$, then $P\left(\frac{B'}{A'}\right) =$

(A) $\frac{3}{5}$ (B) $\frac{5}{8}$ (C) $\frac{3}{8}$ (D) $\frac{5}{6}$

35. If A and B are two events such that $P(A) \ne 0$ and $P\left(\frac{B}{A}\right) = 1$, then :

(A) $P\left(\frac{A}{B}\right) = 1$ (B) $P\left(\frac{B}{A}\right) = 1$

(C) $P\left(\frac{A}{B}\right) = 0$ (D) $P\left(\frac{B}{A}\right) = 0$

36. The angle between the vector $2\vec{i} - 3\vec{j} + 2\vec{k}$ and $\vec{i} - 4\vec{j} + 5\vec{k}$ is given by :

(A) 30° (B) 90° (C) 45° (D) 60°

37. If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ then $a \cdot b + b \cdot c + c \cdot a$ equal to :

(A) 47 (B) –25 (C) 0 (D) 25

38. $\int \log 2\, dx =$

(A) $x + k$ (B) $\log 2 + k$
(C) $x \log 2 + k$ (D) $x^2 \log 2 + k$

39. $\int \log x\, dx =$

(A) $\frac{1}{x} + k$ (B) $x \log x + k$
(C) $x \log x - x + k$ (D) $x \log x + x + k$

40. If $f : R \to R$ defined by $f(x) = 2x + 3$ then $f^{-1}(x) =$

(A) $2x - 3$ (B) $\frac{x-3}{2}$ (C) $\frac{x+3}{2}$ (D) None

41. The function $f(x) = \log(x^2 + \sqrt{x^2+1})$ is :
(A) even function (B) odd function
(C) Both (D) None of these

42. $3\sin^{-1} x = \ldots\ldots, |x| \le \frac{1}{2}$
(A) $\sin^{-1}(4x^3 - 3x)$ (B) $\sin^{-1}(3x + 4x^3)$
(C) $\sin^{-1}(3x - 4x^3)$ (D) $\sin^{-1}(3x^3 - 4x)$

43. $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) =$
(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $-\frac{\pi}{3}$ (D) $-\frac{\pi}{6}$

44. $\vec{i}.\vec{j} =$
(A) 0 (B) 1 (C) $\vec{k}$ (D) $-\vec{k}$

45. $\vec{k} \times \vec{j} =$
(A) 0 (B) 1 (C) $\vec{i}$ (D) $-\vec{i}$

46. $\int \frac{dx}{x} = \ldots\ldots$
(A) $x + k$ (B) $\frac{1}{x^2} + k$ (C) $-\frac{1}{x^2} + k$ (D) $\log x + k$

47. $\int_a^b x^3 dx = \ldots\ldots$
(A) $\frac{b^3 - a^3}{3}$ (B) $\frac{b^4 - a^4}{4}$ (C) $\frac{b^2 - a^2}{2}$ (D) 0

48. $\frac{d}{dx}(\sec x) = ?$
(A) $\sec x \cot x$ (B) $\sec x \tan x$
(C) $\tan x$ (D) $\cot x$

49. $\frac{d}{dx}(\sin 4x) = ?$
(A) $4 \sin 4x$ (B) $4 \cos 4x$
(C) $4x \sin x$ (D) $4x \cos 4x$

50. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then :
(A) $|A| = 0$ (B) A^{-1} exists
(C) A^{-1} does not exist (D) None of these

51. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ then $A^3 =$
(A) 3A (B) 2A (C) 4A (D) A

52. $P(E) =$
(A) $n(E) + n(s)$ (B) $\frac{n(E)}{n(s)}$
(C) $\frac{n(s)}{n(E)}$ (D) $n(E) - n(s)$

53. $P(A) + P(A') =$
(A) 0 (B) 1 (C) -1 (D) $P(E)$

54. The planes $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are :
(A) Perpendicular
(B) Parallel
(C) Intersect along y-axis
(D) Pass through point $\left(0, 0, \frac{5}{4}\right)$

55. $\int \sec^2(3x + 5)\, dx = ?$
(A) $\frac{1}{3}\tan(3x+5)+k$ (B) $-\frac{1}{3}\tan(3x+5)+k$
(C) $\frac{1}{5}\tan(3x+5)+k$ (D) $-\frac{1}{5}\tan(3x+5)+k$

56. $\int \frac{x+\sin x}{1+\cos x} dx$ is equal to :
(A) $\log|1+\cos x|+c$ (B) $\log|x+\sin x|+c$
(C) $x - \tan\frac{x}{2}+c$ (D) $x \cdot \tan\frac{x}{2}+c$

57. The value of the determinant $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$ is :
(A) 0 (B) -1 (C) 2 (D) -2

58. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to :
(A) 6 (B) ± 6 (C) -6 (D) 0

59. Which of the following plane is parallel to the zx–plane ?
(A) $z = k$ (B) $y = k$
(C) $x = k$ (D) None of these

60. The modulus of $x\vec{i} + y\vec{j} + z\vec{k}$ is :
(A) $x^2 + y^2 + z^2$ (B) $\sqrt{x^2+y^2+z^2}$
(C) $\sqrt{x+y+z}$ (D) $\frac{1}{\sqrt{x^2+y^2+z^2}}$

61. $[\vec{x}\ \vec{y}\ \vec{z}] =$
(A) $[\vec{z}\ \vec{y}\ \vec{x}]$ (B) $[\vec{y}\ \vec{x}\ \vec{z}]$
(C) $[\vec{x}\ \vec{z}\ \vec{y}]$ (D) $[\vec{z}\ \vec{x}\ \vec{y}]$

62. The integrating factor (I.F.) of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is :
(A) $\frac{1}{x}$ (B) e^{-x} (C) e^{-y} (D) x

63. $\frac{d}{dx}(\sin^{-1}x + \cos^{-1}x) =$
(A) 0 (B) 1
(C) $\frac{\pi}{2}$ (D) $\frac{1}{\sqrt{1-x^2}}$

64. If $y = \sin(x^3)$, then $\frac{dy}{dx} =$
(A) $x^3 \cos(x^3)$ (B) $3x^2 \sin(x^3)$
(C) $3x^2 \cos(x^3)$ (D) $\cos(x^3)$

65. $\sin^{-1}\frac{1}{2} =$

(A) π (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

66. $\sin(\sin^{-1}x) =$

(A) $\sin x$ (B) $\cos x$

(C) x (D) $\pi - x$

67. $\int e^{2\log\sec x}dx =$

(A) $e^{2\log\sec x} + c$ (B) $\tan x + c$

(C) $\sec x \cdot \tan x + c$ (D) $\log \sec x + c$

68. The position vector of the point (4, 5, 6) is :

(A) $4\vec{i}+5\vec{j}+6\vec{k}$ (B) $4\vec{i}-5\vec{j}-6\vec{k}$

(C) $2\vec{i}+\vec{j}+\vec{k}$ (D) $\vec{i}+\vec{j}+\vec{k}$

69. $|2\vec{i}-3\vec{j}+\vec{k}| =$

(A) 14 (B) $\sqrt{14}$ (C) $\sqrt{3}$ (D) 2

70. $\int\frac{(1+\log x)^2}{x}dx =$

(A) $\frac{1}{3}(1+\log x)^3+c$ (B) $\frac{1}{2}(1+\log x)^2+c$

(C) $\log(\log 1 + x) + 2$ (D) None of these

71. $\int\frac{dx}{x^2+16} = ?$

(A) $\frac{1}{16}\tan^{-1}\frac{x}{16}+k$ (B) $\frac{1}{4}\tan^{-1}\frac{x}{4}+k$

(C) $\frac{1}{4}\tan^{-1}\frac{4}{x}+k$ (D) $\frac{1}{4}\tan^{-1}\frac{16}{x^2}+k$

72. If $\begin{vmatrix} x & 5 \\ 5 & x \end{vmatrix} = 0$ then $x =$

(A) ± 5 (B) 6 (C) 0 (4) 4

73. The solution of the differential equation $\frac{dx}{x}+\frac{dy}{y} = 0$ is :

(A) $xy = k$ (B) $\frac{x}{y} = k$

(C) $x + y = k$ (D) $x - y = k$

74. If A and B are square matrices then $(AB)' =$

(A) B'A' (B) A'B' (C) AB' (D) A'B'

75. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ then *adj A* is :

(A) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

76. If A and B are two events such that $P(A \cup B) = P(A)$, then

(A) $P\left(\frac{B}{A}\right)=0$ (B) $P\left(\frac{A}{B}\right)=0$

(C) $P\left(\frac{A}{B}\right)=1$ (D) $P\left(\frac{B}{A}\right)=1$

77. $P(A \cap B) = 0$ if the event A and B are :

(A) Independent (B) Mutually exclusive

(C) Dependent (D) None of these

78. $\int_0^{\pi/4} \sec^2\theta\, d\theta =$

(A) 0 (B) 1 (C) -1 (D) $\frac{\pi}{4}$

79. $\int e^{3x}dx =$

(A) $k + e^{3x}$ (B) $k - e^{3x}$ (C) $k + 3e^{3x}$ (D) $k+\frac{e^{3x}}{3}$

80. $\frac{d}{dx}\left(\frac{1}{\sin x}+e^x\right) =$

(A) $-\frac{1}{\sin^2 x}+e^x$ (B) $\operatorname{cosec}x + e^x$

(C) $-\operatorname{cosec}x \cot x + e^x$ (D) $\operatorname{cosec}x \cot x + e^x$

81. $\frac{d}{dx}(\log 3^x) =$

(A) $\frac{1}{3^x}$ (B) log3 (C) $x$log3 (D) 1

82. $\sin\left(\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}\right) =$

(A) 0 (B) -1 (C) 1 (D) $\frac{1}{2}$

83. $\cos^{-1}\left(\cos\frac{8\pi}{5}\right) =$

(A) $\frac{8\pi}{4}$ (B) $\frac{12\pi}{4}$ (C) $\frac{2\pi}{5}$ (D) $\frac{4\pi}{5}$

84. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} =$

(A) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

85. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is :

(A) 18 (B) 512

(C) 81 (D) None of these

86. If order of the differential equation $\left(\frac{dy}{dx}\right)^2 + y = x$ is

(A) 0 (B) 1 (C) 2 (D) 3

87. The degree of the equation $\left(\frac{d^2y}{dx^2}\right)^2 - x\left(\frac{dy}{dx}\right)^3 = y^3$ is :

(A) 0 (B) 1 (C) 2 (D) 3

88. If A and B are two independent events, then :

(A) $P(A \cap B) = P(A) \times P(B)$

(B) $P(AB) = 1 - P(A')P(B')$

(C) $P(AB) = 1 + P(A')P(B')$

(D) $P(AB) = \frac{P(A')}{P(B')}$

89. The probability of an event is $\frac{3}{7}$. Then odd against the event is :

(A) 4 : 3 (B) 7 : 3 (C) 3 : 7 (D) 3 : 4

90. The distance of the plane $2x - 3y + 6z + 7 = 0$ from the point (2, –3, –1) is

(A) 4 (B) 3 (C) 2 (D) $\frac{1}{5}$

91. The direction cosines of the normal to the plane $2x - 3y - 6z - 3 = 0$ are

(A) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$ (B) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

(C) $\frac{-2}{7}, \frac{-3}{7}, \frac{-6}{7}$ (D) None of these

92. If $|\vec{a} \times \vec{b}| = |\vec{a} . \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(A) 0 (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) π

93. The modulus of the vector $19\vec{i} + 5\vec{j} - 6\vec{k}$ is :

(A) $\sqrt{322}$ (B) $\sqrt{420}$ (C) $\sqrt{421}$ (D) $\sqrt{422}$

94. $\int \frac{dx}{1 - \sin x} =$

(A) $\tan x - \sec x + k$ (B) $\tan x + \sec x + k$
(C) $\tan x + \sec^2 x + k$ (D) $2(\tan x - \sec x) + k$

95. $\int_a^b x^2 dx =$

(A) $\frac{b^3 - a^3}{3}$ (B) $\frac{a^3 - b^3}{3}$ (C) $\frac{a^2 - b^2}{2}$ (D) $\frac{b^2 - a^2}{2}$

96. $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} =$

(A) –18 (B) 15 (C) –15 (D) 18

97. The value of the determinant having two rows (or columns) identical is :

(A) 1 (B) –1 (C) 0 (D) 2

98. If $y = \log_e \sqrt{x}$, then $\frac{dy}{dx} =$

(A) $\frac{1}{\sqrt{x}}$ (B) $\frac{1}{2\sqrt{x}}$ (C) $\frac{1}{x}$ (D) $\frac{1}{2x}$

99. If $A = \begin{bmatrix} 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & 10 & 9 \\ 8 & 7 & 6 \end{bmatrix}$ then A + B = ...

(A) $\begin{bmatrix} 20 & 20 & 20 \\ 20 & 20 & 20 \end{bmatrix}$ (B) $\begin{bmatrix} 10 & 10 & 10 \\ 10 & 10 & 10 \end{bmatrix}$

(C) $\begin{bmatrix} 10 & 5 & 10 \\ 5 & 10 & 10 \end{bmatrix}$ (D) $\begin{bmatrix} 25 & 10 & 15 \\ 15 & 10 & 25 \end{bmatrix}$

100. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then AB =

(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) 10

SECTION – B : Non-Objective Type Questions

SHORT ANSWER TYPE QUESTIONS

Direction : *Question Nos. 1 to 30 are of short answer type. Answer only 15 questions from these.* $15 \times 2 = 30$

1. **Integrate :** $\int \cot x \log \sin x dx$

2. **Evaluate :** $\int \cot^2 x dx$

3. **Evaluate :** $\int \sin x \sin 2x \sin 3x \, dx$

4. **Evaluate :** $\int \tan^2 x \sec^4 x \, dx$.

5. **Evaluate :** $\int \sec^4 x \, dx$

6. **Evaluate :** $\int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x} dx$

7. **Find** $\int_1^{e^2} \frac{dx}{x(1 + \log x)^2}$

8. **Evaluate :** $\int_0^{\pi/4} \frac{\sin x}{\cos 3x + 3\cos x} dx$

9. **If $f : R \to R$ be a function defined by $f(x) = x^2$ show that the function f is many one into.**

10. **If** $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, **verify that $A^2 = I$.**

11. **If** $\begin{bmatrix} 15 & x+y \\ 2 & y \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ x-y & 3 \end{bmatrix}$, **find the value of x.**

12. **Using properties of determinants, prove the following :**

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc.$$

13. **Differentiate :** $\sec(\tan\sqrt{x})$

14. **If** $y = \sqrt{\cos(1 + x^2)}$, **find** $\frac{dy}{dx}$

15. **If** $y = \sqrt{x^2 + ax + 1}$ **then find** $\frac{dy}{dx}$.

16. **Solve the following differential equation :**

$$\frac{dy}{dx} + \sec x . y = \tan x$$

17. **Solve the following differential equation :**

$$\sin x \frac{dy}{dx} + \cos x . y = \cos x . \sin^2 x.$$

18. **Prove that :** $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2\vec{a} \times \vec{b}$

19. **If** $\vec{a} = 2\hat{i} + 3\hat{k}$ **and** $\vec{b} = 3\hat{i} + 5\hat{k}$ **.Then find** $\vec{a} \times \vec{b}$ **.**

20. **Find the Cartesian equation of the line which passes through the point (–2, 4, –5) and is parallel to the line** $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$

21. **If $P(A) = 0.4$, $P(B) = 0.8$, $P(B/A) = 0.6$. Find $P(A/B)$ and $P(A \cup B)$.**

22. **Prove that :** $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$

23. **Prove that :**

$$\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left|x - \sqrt{(1-y^2)} - y\sqrt{(1-x^2)}\right|$$

24. Show that : $\sin^{-1}\frac{2\sqrt{2}}{3}+\sin^{-1}\frac{1}{3}=\frac{\pi}{2}$

25 Prove that : $3\sin^{-1}x = \sin^{-1}(3x-4x^3)$

26. Differentiate : $\sin^{-1}(\cos x)$

27. Is the function $f : R \to R$ onto function where $f(x) = 2x$? Give reasons.

28. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, then prove that $yz + zx + xy = 1$.

29. Find the value of x from the following :

$$\begin{pmatrix} 2x-y & 5 \\ 3 & y \end{pmatrix}=\begin{pmatrix} 6 & 5 \\ 3 & -2 \end{pmatrix}$$

30. Solve : $\frac{dy}{dx}+1=e^{x+y}$

LONG ANSWER TYPE QUESTIONS

Direction : *Question Nos. 31 to 38 are long answer type question. Answer any 4 questions from these.* $5 \times 4 = 20$

31. If $x + y = \sec^{-1}(x + y)$ then find $\frac{dy}{dx}$

32. At what point, the slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is maximum ? Also, find the maximum slope.

33. Prove that the angle in a semi-circle is a right angle.

34. Find the equation of the line intersecting the line $\frac{x-a}{1}=\frac{y}{1}=\frac{z-a}{1}$ and $\frac{x+a}{1}=\frac{y}{1}=\frac{z+a}{2}$ and parallel to the line $\frac{x-a}{2}=\frac{y-a}{1}=\frac{z-2a}{3}$

35. Four cards are drawn randomly from a well shuffled pack of 52 cards. Find the probability of getting 3 diamonds and one spade.

36. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

37. Solve the following LPP graphically

Minimize $z = 5x + 7y$

subject to $x + y \le 4$

$3x + 8y \le 24$

$10x + 7y \le 35$

$x, y \ge 0$

38. Evaluate : $\int_0^{\pi/2}\frac{\sqrt{\cos x}\,dx}{\sqrt{\sin x}+\sqrt{\cos x}}$

ANSWER WITH EXPLANATIONS

SECTION – A

OMR ANSWER-SHEET

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ANSWER

1. (A)	2. (B)	3. (A)	4. (D)	5. (B)
6. (C)	7. (B)	8. (C)	9. (C)	10. (A)
11. (C)	12. (A)	13. (B)	14. (D)	15. (B)
16. (C)	17. (A)	18. (B)	19. (A)	20. (B)
21. (D)	22. (B)	23. (B)	24. (D)	25. (D)
26. (C)	27. (B)	28. (B)	29. (A)	30. (C)
31. (C)	32. (B)	33. (B)	34. (D)	35. (B)
36. (B)	37. (B)	38. (C)	39. (C)	40. (B)
41. (A)	42. (C)	43. (D)	44. (A)	45. (D)
46. (D)	47. (B)	48. (B)	49. (B)	50. (B)
51. (C)	52. (B)	53. (B)	54. (B)	55. (A)
56. (D)	57. (A)	58. (B)	59. (B)	60. (B)
61. (D)	62. (A)	63. (A)	64. (C)	65. (B)
66. (C)	67. (B)	68. (A)	69. (B)	70. (A)
71. (B)	72. (A)	73. (B)	74. (A)	75. (C)
76. (A)	77. (B)	78. (B)	79. (D)	80. (C)
81. (B)	82. (C)	83. (C)	84. (D)	85. (B)
86. (B)	87. (C)	88. (A)	89. (A)	90. (C)
91. (A)	92. (C)	93. (D)	94. (B)	95. (B)
96. (D)	97. (C)	98. (D)	99. (A)	100. (B)

SECTION – B

1. Let, $\log \sin x = t \Rightarrow \frac{1}{\sin x} \times \cos x = \frac{dt}{dx}$

$$\Rightarrow \cot x = \frac{dt}{dx} \Rightarrow dx = \frac{dt}{\cot x}$$

$$\therefore \int \cot x \log \sin x \, dx = \int \cot x . t \frac{dt}{\cot x}$$

$$= \int t \, dt = \frac{t^2}{2} + C = \frac{(\log \sin x)^2}{2} + C \text{ Ans.}$$

2. Let, $I = \int (\cot^2 x) \, dx$

$$= \int (\operatorname{cosec}^2 x - 1) dx$$

$$= \int \operatorname{cosec}^2 x dx - \int dx$$

$$= -\cot x - x + C; \text{ Ans.}$$

3. Let, $I = \int \sin x \sin 2x \sin 3x \, dx$

$$= \frac{1}{2} \int \sin 2x \, (2 \sin 3x \sin x) \, dx$$

$$= \frac{1}{2} \int \sin 2x \, (\cos 2x - \cos 4x) \, dx$$

$$= \frac{1}{4} \int (2 \sin 2x \cos 2x - 2 \cos 4x \sin 2x) \, dx$$

$$= \frac{1}{4} \int [\sin 4x - (\sin 6x - \sin 2x)] \, dx$$

$$= \frac{1}{4} \int [\sin 2x + \sin 4x - \sin 6x)] \, dx$$

$$= \frac{1}{4} \left(-\frac{\cos 2x}{2} - \frac{\cos 4x}{4} + \frac{\cos 6x}{6} \right) + C$$

$$= \frac{\cos 6x}{24} - \frac{\cos 4x}{16} - \frac{\cos 2x}{8} + C; \text{ Ans.}$$

4. Let, $z = \tan x$, then $dz = \sec^2 x \, dx$

Now, $\int \tan^2 x \sec^4 x \, dx = \int \tan^2 x \sec^2 x \cdot \sec^2 x \, dx$

$$= \int \tan^2 x \, (1 + \tan^2 x) \sec^2 x \, dx$$

$$= \int z^2 (1 + z^2) \, dz = \int (z^2 + z^4) \, dz$$

$$= \frac{z^3}{3} + \frac{z^5}{5} + c$$

$$= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + c \text{ Ans.}$$

5. Let, $z = \tan x$, then $dz = \sec^2 x \, dx$

Now, $\int \sec^4 x \, dx = \int \sec^2 x \cdot \sec^2 x \, dx$

$$= \int (1 + \tan^2 x) \sec^2 x \, dx$$

$$= \int (1 + z^2) \, dx = z + \frac{z^3}{3} + c$$

$$= \tan x + \frac{\tan^3 x}{3} + c \text{ Ans.}$$

6. Let $\cos x = t \Rightarrow -\sin x dx = dt$

$\Rightarrow \sin x dx = -dt$

When $x = 0$, $t = \cos 0 = 1$ and when $x = \frac{\pi}{2}$, $t = \cos \frac{\pi}{2} = 0$

Now, $\int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x} dx = \int_1^0 \frac{-dt}{1 + t^2} = -\frac{1}{1}\left[\tan^{-1} \frac{t}{1}\right]_1^0$

$$= -[\tan^{-1} 0 - \tan^{-1} 1] = -\left(0 - \frac{\pi}{4}\right) = \frac{\pi}{4} \text{ Ans.}$$

7. Let $z = 1 + \log x$, then $dz = \frac{1}{x} dx$

Also when $x = 1$, $z = 1$

When $x = e^2$, $z = 1 + \log e^2 = 1 + 2 \log e = 1 + 2 = 3$

Now, $I = \int_1^{e^2} \frac{dx}{x(1 + \log x)^2} = \int_1^3 \frac{dz}{z^2} = \int_1^3 z^{-2} dz$

$$= \left[\frac{Z^{-1}}{-1}\right]_1^3 = \left[-\frac{1}{Z}\right]_1^3 = -\left[\frac{1}{3} - \frac{1}{1}\right] = \frac{2}{3} \text{ Ans.}$$

8. $\int \frac{\sin x}{\cos 3x + 3 \cos x}$

$$= \int \frac{\sin x}{(4 \cos^3 x - 3 \cos x) + 3 \cos x} . dx$$

$$= \int \frac{\sin x}{4 \cos^3 x} dx = \frac{1}{4} \int \tan x . \sec^2 x . dx$$

$$= \frac{1}{4} . \frac{1}{2} \int \tan^{-2} x . dx = \frac{1}{8} \int \tan^2 x \, dx$$

[Putting $z = \tan x$]

$$\therefore \int_0^{\pi/4} \frac{\sin x}{\cos 3x + 3 \cos x} dx$$

$$= \frac{1}{8} [\tan^2 x]_0^{\pi/4}$$

$$= \frac{1}{8}\left[\tan^2\frac{\pi}{4} - \tan^2 0\right] = \frac{1}{8}[1-0] = \frac{1}{8}$$

$$\therefore \int_0^{\pi/4} \frac{\sin x}{\cos 3x + 3\cos x} . dx = \frac{1}{8} \text{ Ans.}$$

9. $f : R \to R$, given by $f(x) = x^2$

(a) f is one-one since $f(-1) = f(1) = 1$

-1 and 1 have the same image.

i.e., f is not injective.

(b) $-2 \in$ codomians R of f but $\sqrt{-2}$ does not belong to domain R of f.

$\therefore$ f is not into i.e., f is many one into function.

10.
$$A^2 = A . A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+1 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 1+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ $A^2 = 1$

11. We have $\begin{bmatrix} 15 & x+y \\ 2 & y \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ x-y & 3 \end{bmatrix}$

By the def. of equality of two matrices, we have

$y = 3$

and $x + y = 8$

$\therefore$ $x + 3 = 8$

$\Rightarrow$ $x = 8 - 3 = 5$

12.
$$\text{LHS} = \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$

operate : $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix}$$

operate : $R_3 \to R_3 - 2R_1$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix}$$

Expand by C_1

$$= (a+b+c) . 1 \begin{vmatrix} b-c & c-a \\ c+a-2b & a+b-2c \end{vmatrix}$$

operate : $R_2 \to R_2 + 2R_1$

$$= (a+b+c)\begin{vmatrix} b-c & c-a \\ a-c & b-a \end{vmatrix}$$

$$= (a+b+c)\,[(b-c)(b-a) - (a-c)(c-a)]$$
$$= (a+b+c)\,[(b^2 - ab - bc + ac) + (a^2 + c^2 - 2ca)]$$
$$= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
$$= a^3 + b^3 + c^3 - 3abc = \text{RHS}$$ **Proved.**

13. Let $y = \sec(\tan\sqrt{x})$

Then $\frac{dy}{dx} = \frac{d(\sec(\tan\sqrt{x}))}{dx}$

$$= \frac{d\{\sec(\tan\sqrt{x})\}}{d\tan\sqrt{x}} \cdot \frac{d(\tan\sqrt{x})}{d\sqrt{x}} \cdot \frac{d\sqrt{x}}{dx}$$

$$= \sec(\tan\sqrt{x})\tan(\tan\sqrt{x}) \cdot \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{\sec^2\sqrt{x}}{2\sqrt{x}}\sec(\tan\sqrt{x}) \cdot \tan(\tan\sqrt{x}) \text{ Ans.}$$

14.
$$\frac{dy}{dx} = \frac{d\sqrt{\cos(1+x^2)}}{d\cos(1+x^2)} \cdot \frac{d\cos(1+x^2)}{d(1+x^2)} \cdot \frac{d(1+x^2)}{dx}$$

$$= \frac{1}{2\sqrt{\cos(1+x^2)}}\{-\sin(1+x^2)\}2x$$

$$= -\frac{x\sin(1+x^2)}{\sqrt{\cos(1+x^2)}} \text{ Ans.}$$

15. $\because Y = \sqrt{x^2 + ax + 1}$

Differential w.r.t. x.

or, $\frac{dy}{dx} = \frac{d\sqrt{x^2+ax+1}}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{d\sqrt{x^2+ax+1}}{d(x^2+ax+1)} \times \frac{d(x^2+ax+1)}{dx}$$

or $\frac{dy}{dx} = \frac{1}{2\sqrt{x^2+ax+1}} \times (2x+a) = \frac{2x+a}{2\sqrt{x^2+ax+1}}$ **Ans.**

16. The given D.E. is

$$\frac{dy}{dx} + \sec x . y = \tan x \quad \text{... (i)}$$

This is a linear D.E.

On compairing by, $\frac{dy}{dx} + Py = Q$

Here, $P = \sec x$ and $Q = \tan x$

Now I.F. $= e^{\int P dx} = e^{\int \sec x\, dx}$

$= e^{\log(\sec x + \tan x)} = \sec x + \tan x$

$\therefore$ The sol. of (i) is

$$y(\sec x + \tan x) = \int \tan x(\sec x + \tan x)\,dx + c$$

$$= \int (\sec x \tan x + \tan^2 x)dx + c$$

$$= \int \sec x \tan x dx + \int \sec^2 x dx - \int 1.dx + c$$

$$= \sec x + \tan x + c$$

17. The given D.E. is

$$\frac{dy}{dx} + \cot x.y = \cos x.\sin x \quad \text{... (i)}$$

This is a linear D.E.

On compairing by : $\frac{dy}{dx} + Py = Q$

Here $P = \cot x$ and $Q = \sin x \cos x$.

I.F. $= e^{\int P dx} = e^{\int \cot x\, dx} = e^{\log \sin x} = \sin x$

∴ The reqd. sol. of (i) is

$$y \sin x = \int \cos x.\sin x.\sin x dx + c$$

$$= \int \sin^2 x.\cos x dx + c$$

$$= \frac{\sin^3 x}{3} + c \quad \text{(On putting } \sin x = t\text{)}$$

$$\Rightarrow y = \frac{1}{3}\sin^2 x + c \operatorname{cosec} x.$$

18. L.H.S. $= (\bar{a} - \bar{b}) \times (\bar{a} + \bar{b})$

$= \bar{a} \times (\bar{a} + \bar{b}) - \bar{b} \times (\bar{a} + \bar{b})$

$= \bar{a} \times \bar{a} + \bar{a} \times \bar{b} - \bar{b} \times \bar{a} - \bar{b} \times \bar{b}$

$= \bar{0} + \bar{a} \times \bar{b} + \bar{a} \times \bar{b} - \bar{0}$ $[\because \bar{a} \times \bar{a} = \bar{0} \text{ or } \bar{b} \times \bar{b} = \bar{0}]$

$= 2(\bar{a} \times \bar{b})$ = R.H.S. **Proved**

19. $$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 3 \\ 3 & 5 & 0 \end{vmatrix}$$

$$= \hat{i}(0-15) - \hat{j}(0-9) + \hat{k}(10-0)$$

$$= -15\hat{i} + 9\hat{j} + 10\hat{k}$$ **Ans.**

20. d.r.s. of the line parallel to

$$\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6} \quad \text{... (1)}$$

ard 3, –5, 6.

∴ Cartesian Eq. of the lines thro' (–2, 4, –5) and parallel to the line (1) is

$$\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$$

21. ∴ $P(A \cap B) = P(A) \cdot P(B/A) = (0.4)(0.6) = 0.24$

Now $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{0.8} = 0.3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.8 - 0.24 = 0.96$

22. Let, $\tan^{-1}x = \alpha$ and $\tan^{-1}y = \beta$

∴ $x = \tan\alpha$ and $y = \tan\beta$

Now, $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\Rightarrow \tan(\alpha - \beta) = \frac{x-y}{1+xy}$

$\Rightarrow \alpha - \beta = \tan^{-1}\frac{x-y}{1+xy}$

∴ $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$. **Proved**

23. ∵ $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$= \sin A\sqrt{1-\sin^2 B} - \sin B\sqrt{1-\sin^2 A}$

Let, $\sin A = x \Rightarrow A = \sin^{-1}x$ and $\sin B = y \Rightarrow B = \sin^{-1}y$

∴ $\sin(A - B) = x\sqrt{(1-y^2)} - y\sqrt{1-x^2}$

$\Rightarrow (A - B) = \sin^{-1}[x\sqrt{1-y^2} - y\sqrt{1-x^2}]$

∴ $\sin^{-1}x - \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} - y\sqrt{1-x^2}]$. **Proved**

24. $$\sin^{-1}\frac{2\sqrt{2}}{3} + \sin^{-1}\frac{1}{3}$$

$$= \sin^{-1}\left\{\frac{2\sqrt{2}}{3}\sqrt{1-\left(\frac{1}{3}\right)^2} + \frac{1}{3}\sqrt{1-\left(\frac{2\sqrt{2}}{3}\right)^2}\right\}$$

$$= \sin^{-1}\left\{\frac{2\sqrt{2}}{3}\sqrt{1-\frac{1}{9}} + \frac{1}{3}\sqrt{1-\frac{8}{9}}\right\}$$

$$= \sin^{-1}\left\{\frac{2\sqrt{2}}{3} \times \sqrt{\frac{8}{9}} + \frac{1}{3} \times \sqrt{\frac{1}{9}}\right\}$$

$$= \sin^{-1}\left\{\frac{2\sqrt{2}}{3} \times \frac{2\sqrt{2}}{3} + \frac{1}{3} \times \frac{1}{3}\right\}$$

$$= \sin^{-1}\left(\frac{8}{9} + \frac{1}{9}\right) = \sin^{-1}1 = \frac{\pi}{2}$$

25. Let, $\sin^{-1}x = \theta$, then $x = \sin\theta$

∵ $\sin 3\theta = 3\sin\theta - 4\sin^3\theta = 3x - 4x^3$

$\Rightarrow 3\theta = \sin^{-1}(3x - 4x^3)$

∴ $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$ **Proved**

26. Let $y = y \sin^{-1}(\cos x)$

Put $u = \cos x$ so that $y = \sin^{-1}u$

∴ $\frac{du}{dx} = -\sin x$ and $\frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}$

Now $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$ [By chain rule]

$$=\frac{1}{\sqrt{1-u^2}}\cdot(-\sin x)=\frac{-\sin x}{\sqrt{1-\cos^2 x}}$$

$$=\frac{-\sin x}{\sin x}=-1$$ **Ans.**

27. $\because$ $f: R\to R$

Here, domain = R, co-domain = R

$y=f(x)=2x$

$\therefore$ x = domain = R

$\therefore$ $-\infty<n<\infty$

$-\infty<2n<\infty$

$-\infty<y<\infty$

$\therefore$ Range = $(-\infty,\infty)=R$ = co-domain

$\therefore$ f is onto function

28. $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$

$\Rightarrow$ $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{2}-\tan^{-1}z$

$\Rightarrow$ $\tan^{-1}\left(\frac{x+y}{1-xy}\right)=\cos^{-1}z$

$\Rightarrow$ $\tan^{-1}\left(\frac{x+y}{1-xy}\right)=\tan^{-1}\frac{1}{z}$

$\Rightarrow$ $\frac{x+y}{1-xy}=\frac{1}{z}$

$\therefore$ $xy+yz+zx=1$ **Proved.**

29. $\begin{pmatrix}2x-y & 5\\ 3 & y\end{pmatrix}=\begin{pmatrix}6 & 5\\ 3 & -2\end{pmatrix}$

Given matrix will equal when corresponding elements are equal on both

So, $y=-2$ and $2x-y=6$

$\Rightarrow$ $2x=6+y=6+(-2)=4$

$\Rightarrow$ $x=\frac{4}{2}=2$ **Ans.**

30. Here $\frac{dy}{dx}+1=e^{x+y}$ ...(i)

Putting $x+y=v$

D.w.r. to x

$$1+\frac{dy}{dx}=\frac{dv}{dx}$$

Putting in eq. (i), we have

$$\frac{dv}{dx}=e^{v}\Rightarrow dx=e^{-v}dv$$

Integrating, we have $\int dx=\int e^{-v}dv$

$\Rightarrow$ $x=-e^{-v}+c$

$\Rightarrow$ $x+e^{-(x+y)}=c$ (where c is constant) **Ans.**

31. $\sec(x+y)=x+y$

Diff.w.r. to x

$$\sec(x+y)\tan(x+y)\left[1+\frac{dy}{dx}\right]=1+\frac{dy}{dx}$$

$$\sec(x+y)\cdot\tan(x+y)+\frac{dy}{dx}\sec(x+y)\tan(x+y)=1+\frac{dy}{dx}$$

$$\frac{dy}{dx}\,|\sec(x+y)\cdot\tan(x+y)-1|=1-\sec(x+y)\tan(x+y)$$

$$\frac{dy}{dx}=\frac{1-\sec(x+y)\tan(x+y)}{\sec(x+y)\tan(x+y)-1}$$

$$\frac{dy}{dx}=\frac{-[\sec(x+y)\tan(x+y)-1]}{[\sec(x+y)\tan(x+y)-1]}$$

$$\Rightarrow\frac{dy}{dx}=-1$$

32. We have $y=-x^3+3x^2+9x-27$

$\therefore$ $\frac{dy}{dx}=-3x^2+6x+9$ = Slope of tangent to the curve

Now, $\frac{d^2y}{dx^2}=-6x+6$

For $\frac{d}{dx}\left(\frac{dy}{dx}\right)=0$

$6x+6=0$

$\Rightarrow$ $x=\frac{-6}{-6}=1$

$\therefore$ $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)=-6<0$

So, the slope of tangent to the curve is maximum, when $x=1$

For $x=1$, $\left(\frac{dy}{dx}\right)_{(x=1)}=-3.1^2+6.1+9=12$

which is maximum slope

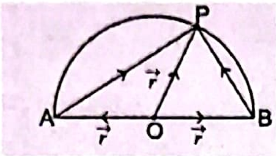
Also, for $x=1$, $y=-1^3+3.1^2+9.1-27$

$=-1+3+9-27$

$=-16$

So, the required point is $(1,-16)$. **Ans.**

33. Let O be the centre of the semi-circle with AOB as its diameter. Let P be a point on the circle, so that $\angle APB$ is an angle in the semi-circle. Join OP. Let O be taken as origin.



Let the position vectors of A, B and P be $\vec{\alpha}, -\vec{\alpha}$ and $\vec{r}$ respectively.

Clearly, $OA = OB = OP$

Now $\vec{AP} = (\vec{r}-\vec{\alpha})$ and $\vec{BP} = (\vec{r}+\vec{\alpha})$

$\therefore \quad \vec{AP}\cdot\vec{BP} = (\vec{r}-\vec{\alpha})\cdot(\vec{r}+\vec{\alpha}) = r^2 - \alpha^2 = OP^2 - OA^2 = 0$

[$\because OP = OA$]

$\therefore \quad AP \perp BP$ i.e. $\angle APB = 90°$ **Proved.**

34. $\because \quad \frac{x-a}{1} = \frac{y}{1} = \frac{z-a}{1} = r$ (say) ...(i)

and $\frac{x+a}{1} = \frac{y}{1} = \frac{z+a}{2} = \lambda$ ...(ii)

Any point on the line (i) is $P(r + a, r, r + a)$

Any point on the line (ii) is $Q(\lambda - a, \lambda, 2\lambda - a)$

Line (i) and (ii) will intersect iff P and Q coincide for same value of λ and r.

$\therefore \quad r + a = \lambda - a \Rightarrow r - \lambda = -2a$ ...(iii)

$r + \lambda \Rightarrow r - \lambda = 0$ ...(iv)

and $r + a = 2\lambda - a \Rightarrow r - 2\lambda = -2a$ ...(v)

Solving (iii) and (iv), we get $\lambda = a$ and $r = a$. $P \equiv (2a, a, 2a)$

Required equation is $\frac{x-2a}{2} = \frac{y-a}{1} = \frac{z-2a}{3}$. **Ans.**

35. 3 diamonds are to be drawn out of 13 diamond cards and 1 spade out of 13 spade cards.

The reqd. prob.

$$= \frac{^{13}C_3 \times {^{13}C_1}}{^{52}C_4} = \frac{13\times12\times11}{3.2.1} \times \frac{13}{1} \times \frac{4.3.2.1}{52.51.50.49}$$

$= \frac{286}{20825}$ **Ans.**

36. $$\Delta = \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}$$

$[C_1 \to C_2 + C_2 + C_3]$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

[Taking $2(a + b + c)$ common from first column]

$$= 2(a+b+c)\begin{vmatrix} 0 & -(b+c+a) & 0 \\ 0 & (b+c+a) & -(a+b+c) \\ 1 & a & c+a+2b \end{vmatrix}$$

$[R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3]$

$$= 2(a+b+c)(a+b+c)^2\begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & a & c+a+2b \end{vmatrix}$$

[Taking $(a + b + c)$ common from first and second rows]

$= 2(a + b + c)^3\, 1\{(-1)(-1) - 1 \times 0\} = 2(a + b + c)^3$.

Proved

37. $z = 5x + 7y$

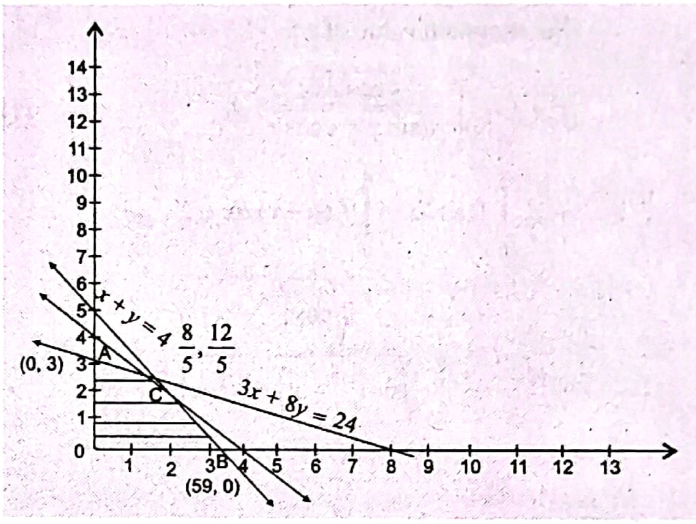
and $x + y \le 4$

consider the equation

$x + y = 4$

x	0	4
y	4	0

Now, $x = 0, y = 0$



$\Rightarrow \quad 0 + 0 \le 4$ is true

$\therefore \quad 0\,(0, 0)$ lines in the region $x + y \le 4$

$3x + 8y \le 24$

Consider the equation

$3x + 8y = 24$

x	0	8
y	3	0

Now, $x = 0, y = 0$

$3 \times 0 + 8 \times 0 \le 24$ is true

$\therefore \quad 0\,(0, 0)$ lies in the region $3x + 8y \le 24$

$10x + 7y \le 35$

Consider the equation

$10x + 7y = 35$

x	0	3.5
y	5	0

Now, $x = 0, y = 0$

$10 \times 0 + 7 \times 0 \le 35$ is true

$\therefore$ 0 (0,0) lies in the region $10x + 7y \le 35$

on solving $x + y = 4$ and $3x + 8y = 24$,

we get the point $\left(\frac{8}{5}, \frac{12}{5}\right)$

on solving $x + y = 4$ and $10x + 7y = 35$,

we get the point $\left(\frac{7}{3}, \frac{5}{3}\right)$

on solving $3x + 8y = 24$ and $10x + 7y = 35$,

we get the point $\left(\frac{112}{59}, \frac{135}{59}\right)$

Value of $z = 5x + 7y$

At A (0, 3) it is $5 \times 0 + 7 \times 3 = 21$

At B (3.5, 0) it is $5 \times 3.5 + 7 \times 0 = 17.5$

At C $\left(\frac{8}{5}, \frac{12}{5}\right)$ it is $5 \times \frac{8}{5} + 7 \times \frac{12}{5} = 24.8$

So minimum value of z is 17.5 **Ans.**

38. Let $I = \int_0^{\pi/2} \frac{\sqrt{\cos x}\,dx}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$ ... (1)

Use $\int_0^{\pi} f(x)\,dx = \int_0^{a} f(a-x)\,dx$ we get

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}\,dx}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}}$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx \quad \text{... (2)}$$

Adding (1) and (2), we get

$$2I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx + \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx$$

$$= \int_0^{\pi/2} dx = [x]_0^{\pi/2} = \frac{\pi}{2}$$

$\therefore \quad I = \frac{\pi}{4}$ **Ans.**

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